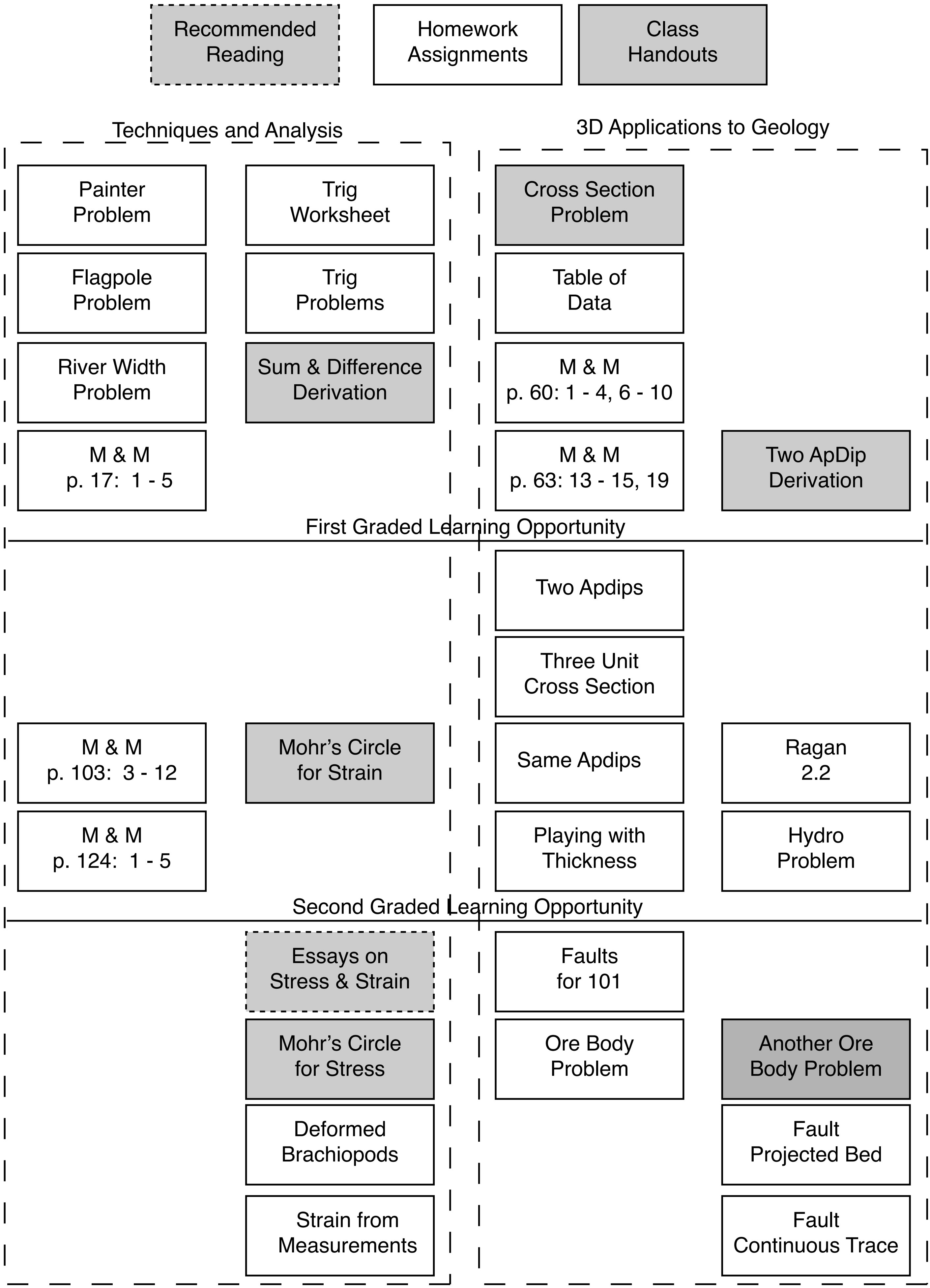

Recommended Reading
Homework Assignments
Class Handouts
Techniques and Analysis
3D Applications to Geology
Painter Problem
Trig Worksheet
Cross Section Problem
Flagpole Problem
Trig Problems
Table of Data
River Width Problem
Sum & Difference Derivation
M & M p. 60: 1 - 4, 6 - 10
M & M p. 17: 1 - 5
M & M p. 63: 13 - 15, 19
Two ApDip Derivation
First Graded Learning Opportunity
Two Apdips
Three Unit Cross Section
M & M p. 103: 3 - 12
Mohr's Circle for Strain
Same Apdips
Ragan 2.2
M & M p. 124: 1 - 5
Playing with Thickness
Hydro Problem
Second Graded Learning Opportunity
Essays on Stress & Strain
Faults for 101
Mohr's Circle for Stress
Ore Body Problem
Another Ore Body Problem
Deformed Brachiopods
Fault Projected Bed
Strain from Measurements
Fault Continuous Trace

It takes three painters four hours to paint seven walls.

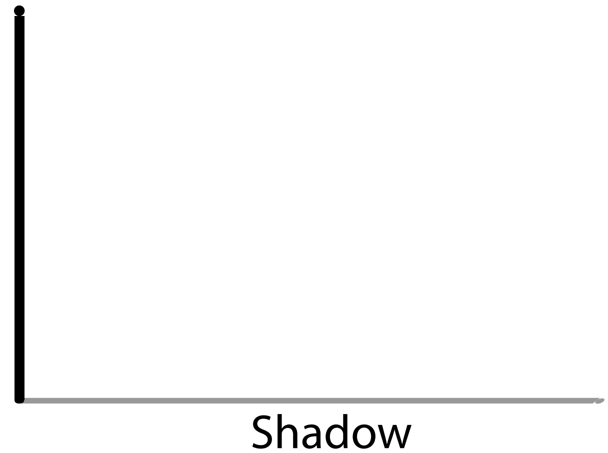
a. How many hours will it take two painters to paint three walls?

b. How many painters would you need to paint 171 walls in 40 hours?

The Height of a Flagpole

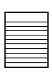
You wish to determine the height of a flagpole standing in the middle of a horizontal grassy, meadow. It is too slippery to climb, there are no ropes attached, but it is a sunny day, and you do have a meter stick. How do you proceed?

Meter Stick



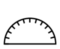
How might you modify your approach if the day were cloudy, but in addition to your meter stick you had a small ruler, and a pad of paper?

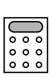
Meter Stick

 Paper, pencil, small ruler

How might you modify your cloudy day approach if you had no pencil or paper, but you had a meter stick, a protractor and a calculator?

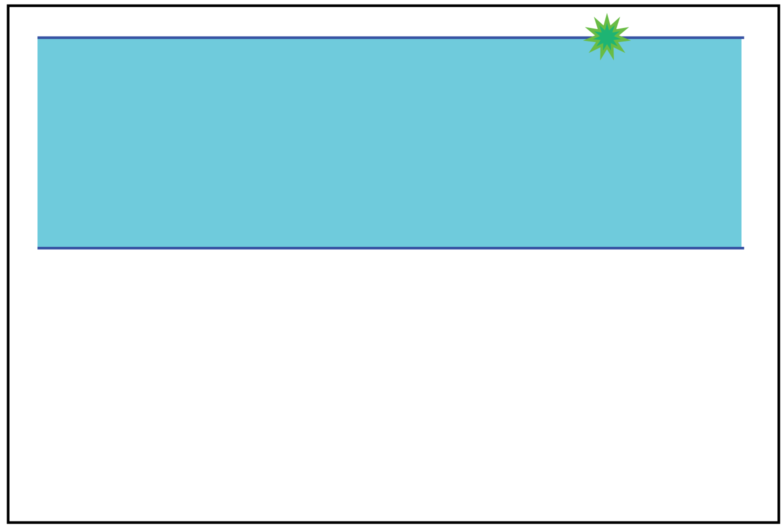
Meter Stick

 Protractor

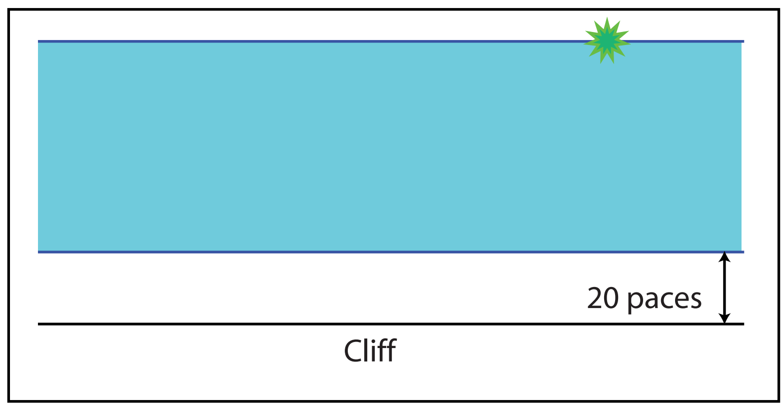
 Calculator

The Width of a River

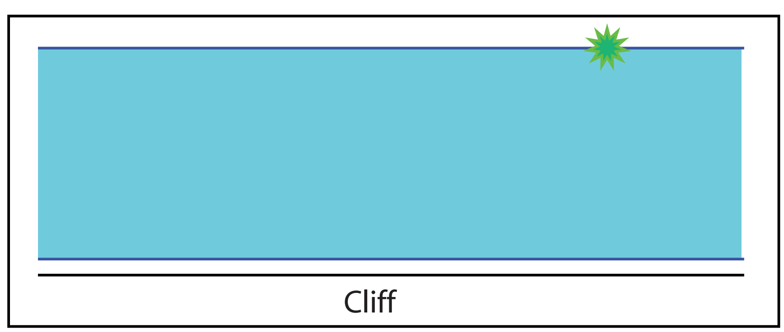
You wish to determine the number of paces it would be across a river which cuts through flat, grassy terrain, there is a tree on the far shore and you have a several stakes you can drive into the ground. You haven't yet learned how to walk on water, and it is July so the river is not frozen. How do you proceed?

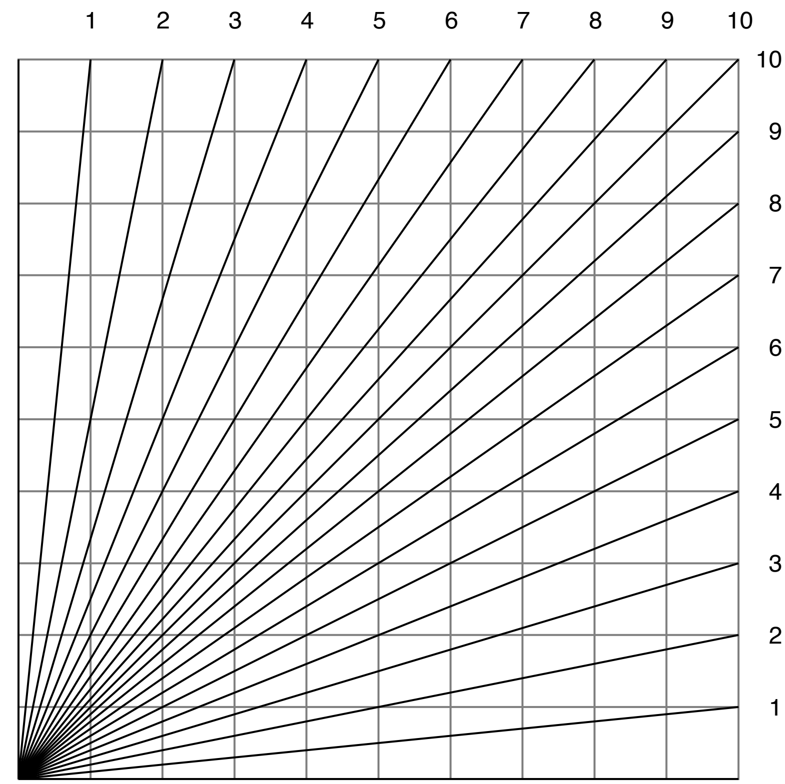


How might you modify your approach if there were a cliff, 20 paces from the bank of the river, on your side:?



How might you modify your approach if there were just enough room to walk between the cliff and the riverbank, but you had a protractor and a calculator?



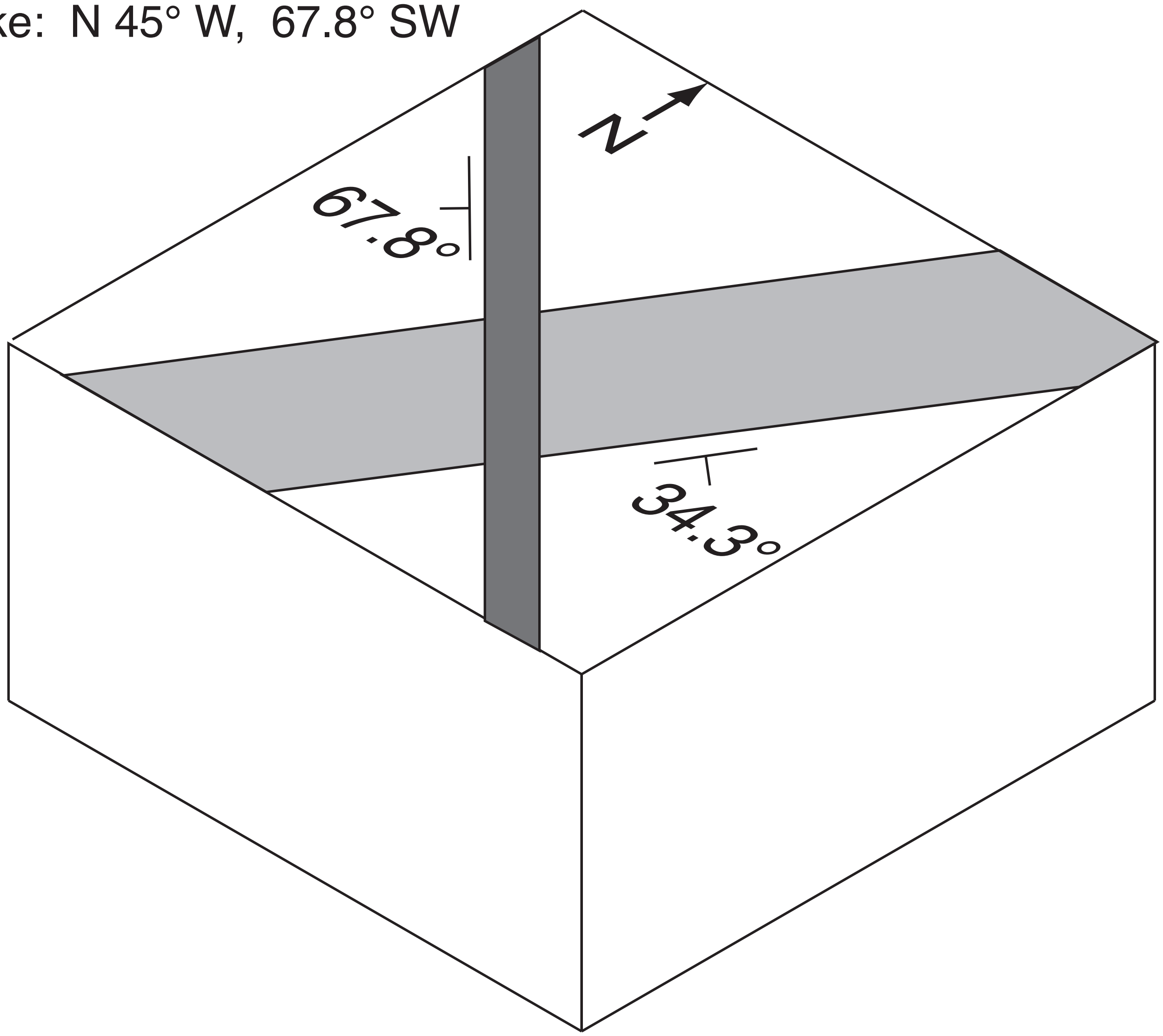


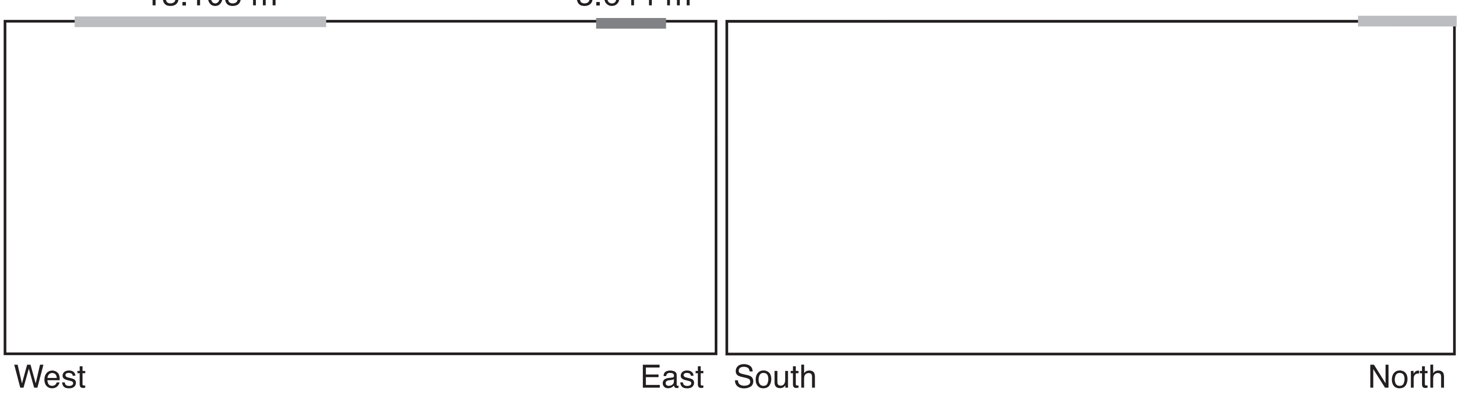
Angle	Opposite	Adjacent	Hypoteneuse	Sine	Cosine	Tangent

In Class Cross Section Problem

Bed: N 32.2° E, 34.3° SE

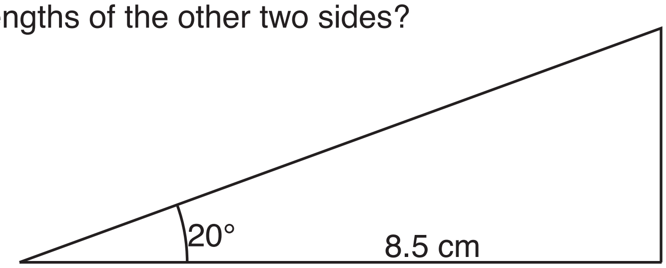
Dike: N 45° W, 67.8° SW



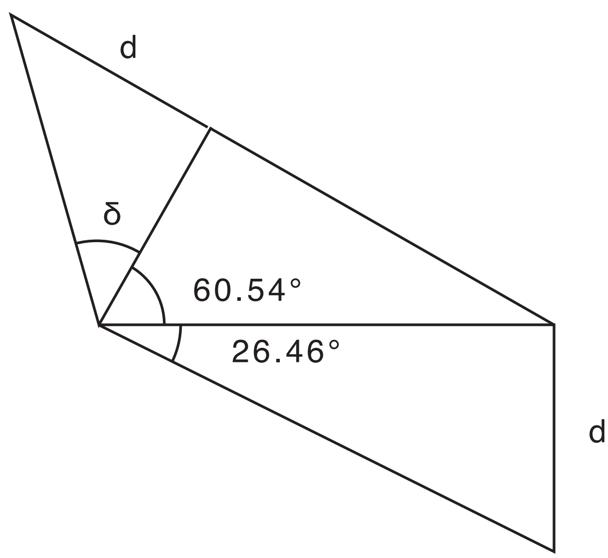


Simple Trig Problems

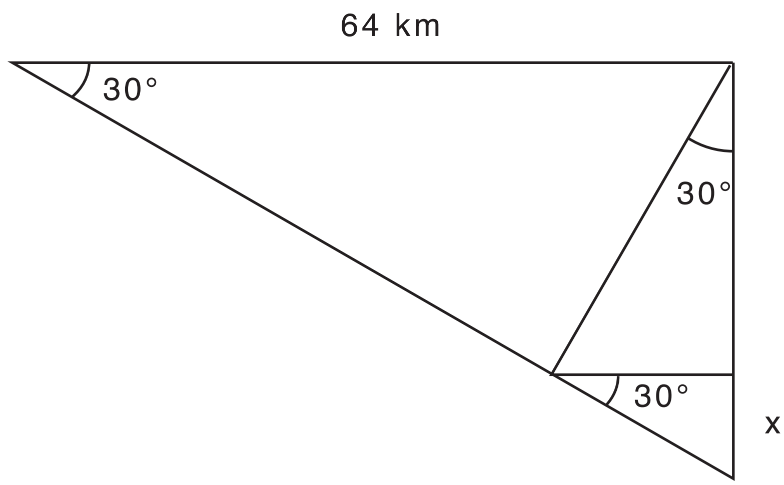
1. What are the lengths of the other two sides?



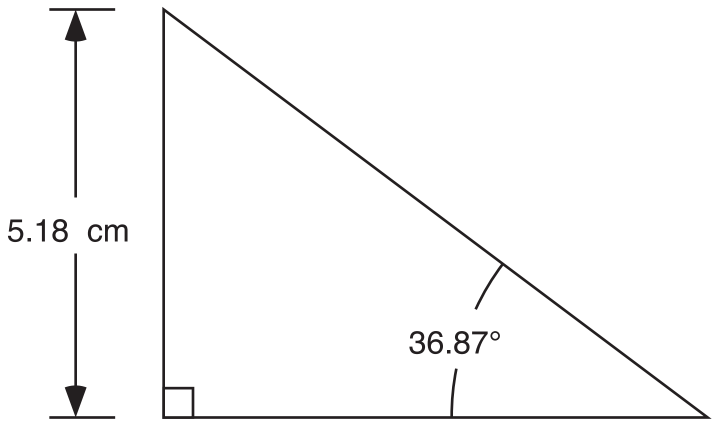
2. What is δ?

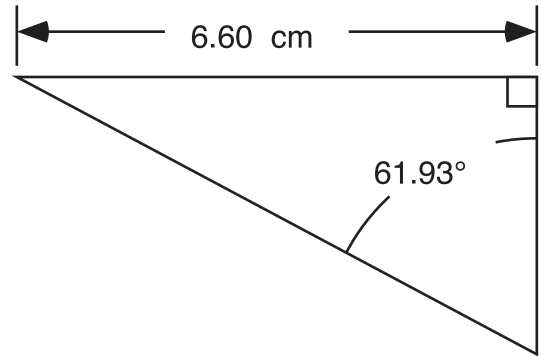


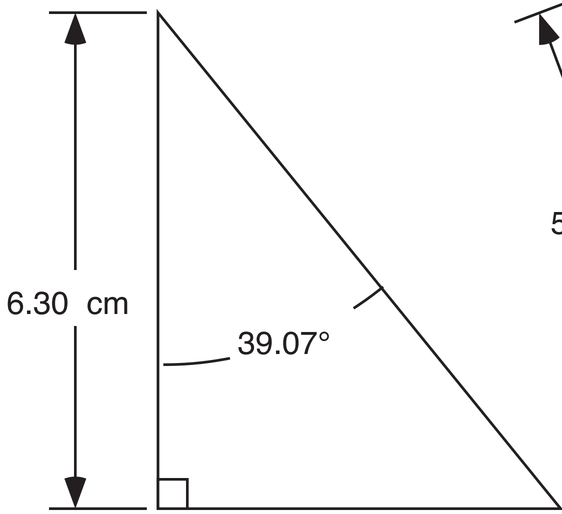
3. What is x?

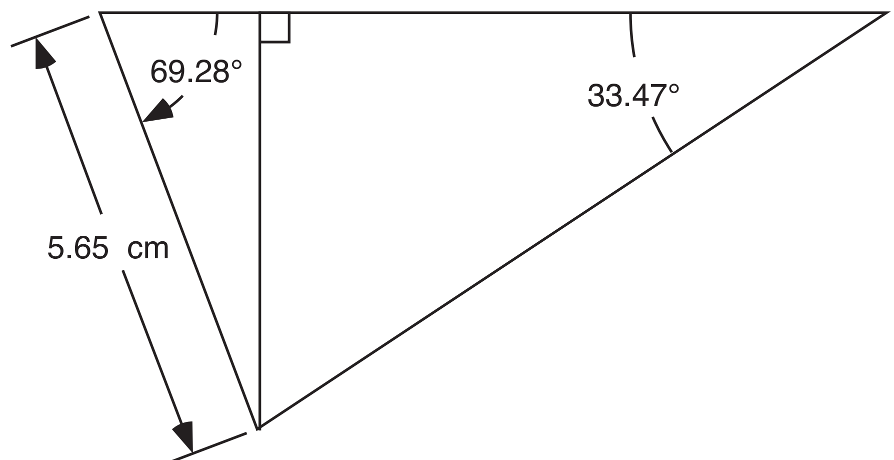


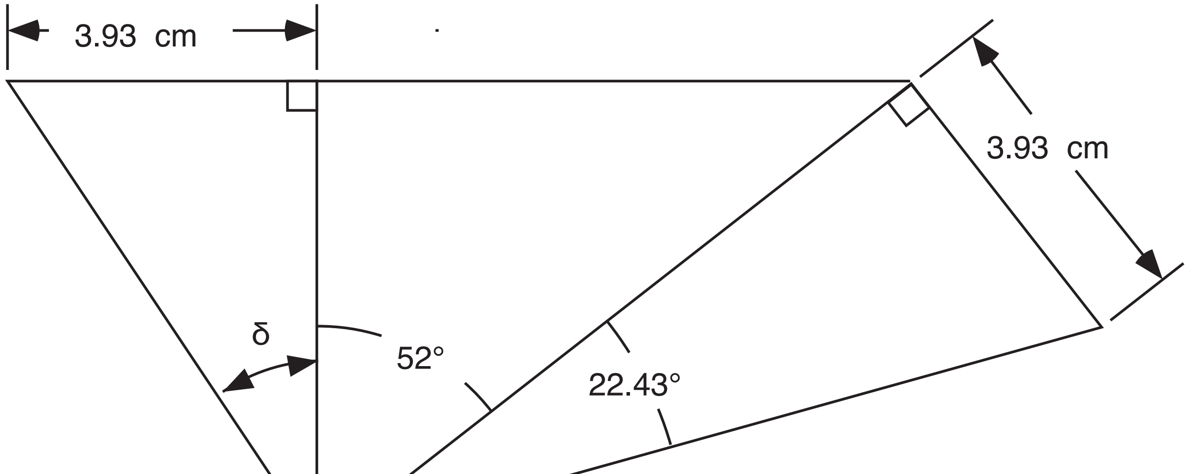
Determine the lengths of all sides of all triangles.



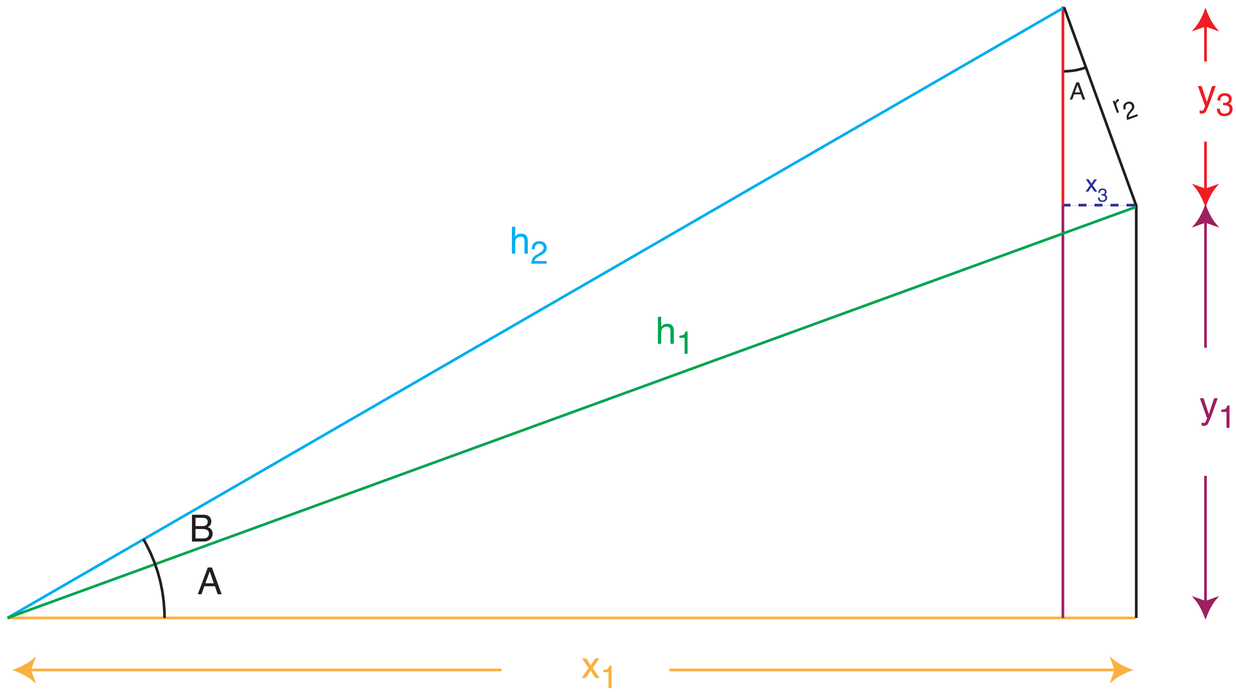








What is δ?



$$\sin(A+B) = \frac{y_1 + y_3}{h_2} = \frac{h_1 \sin A + r_2 \cos A}{h_2} = \frac{h_2 \cos B \sin A + h_2 \sin B \cos A}{h_2}$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

Note that $\sin(-A) = -\sin(A)$ and that $\cos(-A) = \cos(A)$

to get

$$\sin(A-B) = \sin[A + (-B)] = \sin A \cos B + [(-\sin B) \cos A] = \sin A \cos B - \sin B \cos A$$

$$\cos(A+B) = \frac{x_1 - x_3}{h_2} = \frac{h_1 \cos A - r_2 \sin A}{h_2} = \frac{h_2 \cos B \cos A - h_2 \sin B \sin A}{h_2}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

Note that $\sin(-A) = -\sin(A)$ and that $\cos(-A) = \cos(A)$

to get

$$\cos(A-B) = \cos[A + (-B)] = \cos A \cos B - [(-\sin B) \sin A] = \cos A \cos B + \sin A \sin B$$

For the case where $A = B = \theta$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

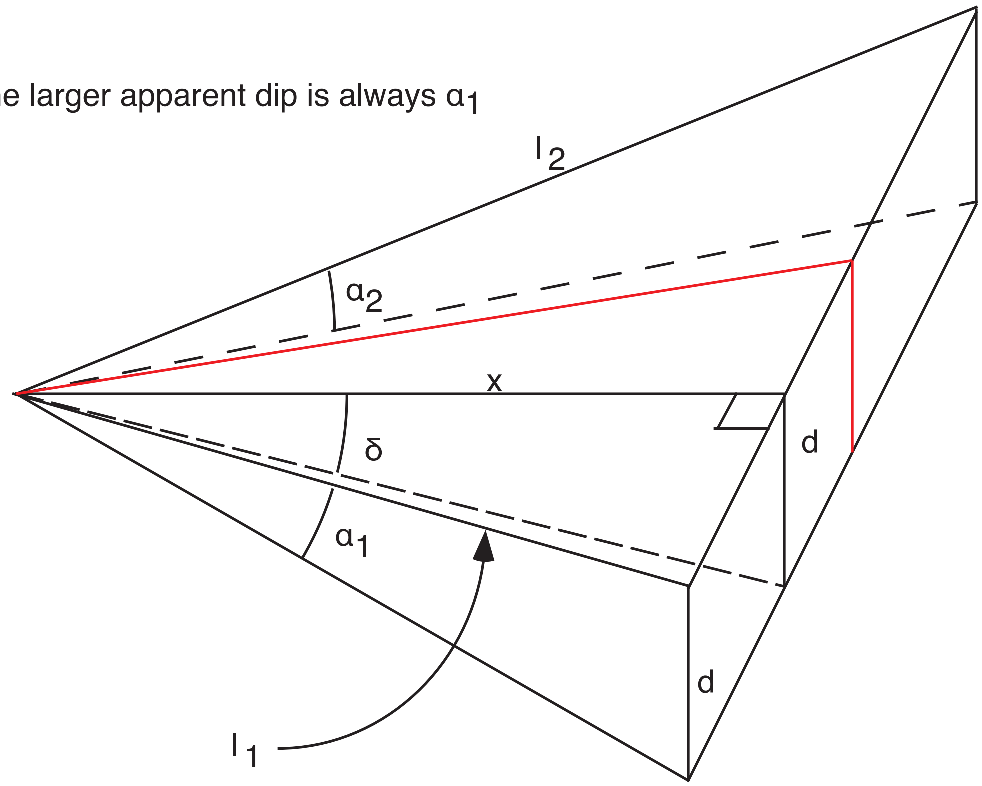
Table of Data

Fill in the empty spaces below with the correct results.
(Show all your work on attached pages.)

	Strike	Dip	Direction of ApDip #1	Magnitude of ApDip #1	Direction of ApDip #2	Magnitude of ApDip #2
1.	N 75 E	22 N	N 50 E		N 50 W	
2.	N 90 E		N 47 E	33	N 0 E	
3.			N 80 W	20	N 10 E	40
4.			N 60 E	30	S 45 E	50
5.			N 78 W	6	N 36 W	25
6.	N 30 W		N 90 E		N 45 E	30
7.	N 26 E	37 S	S 26 W		S 64 E	

Two Apparent Dip Derivation

Note that the larger apparent dip is always α_1

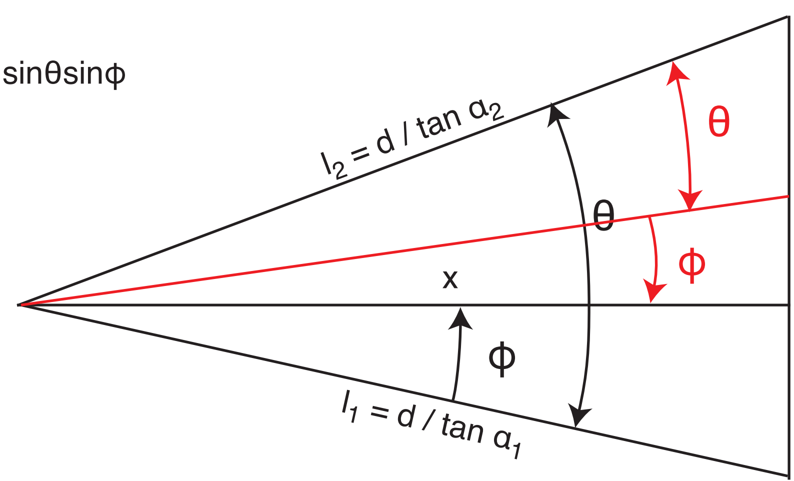


A: $x / l_2 = \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

B: $x / l_1 = \cos\phi$

divide equation A by equation B to get:

$$l_1 / l_2 = \cos\theta + \sin\theta\tan\phi$$

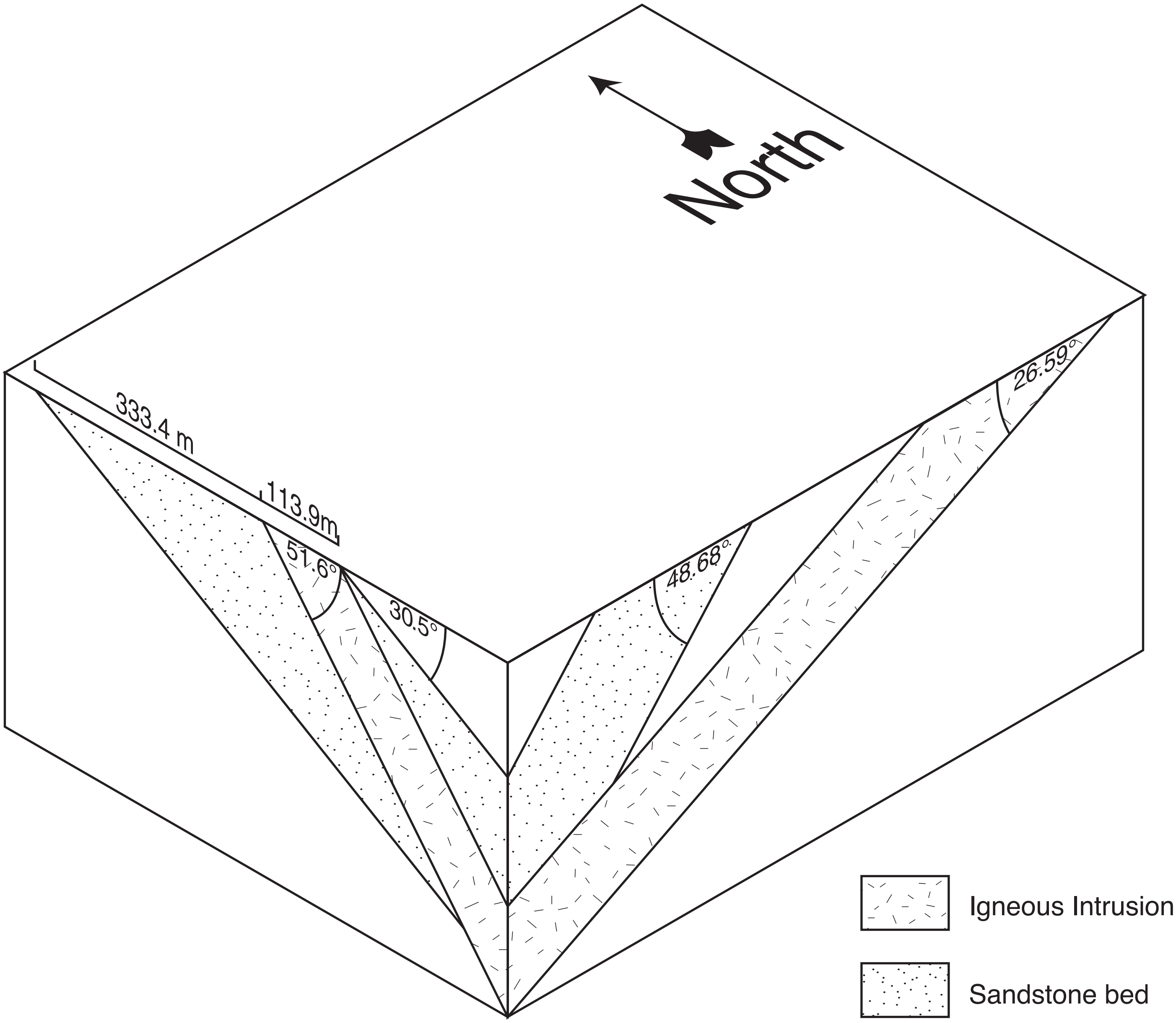


$$\tan\phi = \frac{\dfrac{\tan \alpha_2}{\tan \alpha_1} - \cos\theta}{\sin\theta}$$

$$\text{So, } \phi = \arctan\left[\frac{\dfrac{\tan \alpha_2}{\tan \alpha_1} - \cos\theta}{\sin\theta}\right]$$

The sign issue is an important one. If φ is negative, it should be measured from l_1 in a direction which is away from, not towards, l_2. This is shown with the red sketches above.

Once we have determined φ we see that $x = d \cos\phi / \tan \alpha_1$ and $\tan \delta = d / x = \tan \alpha_1 / \cos\phi$



1. What is the attitude of the igneous intrusion?

2. What is the attitude of the sandstone bed?

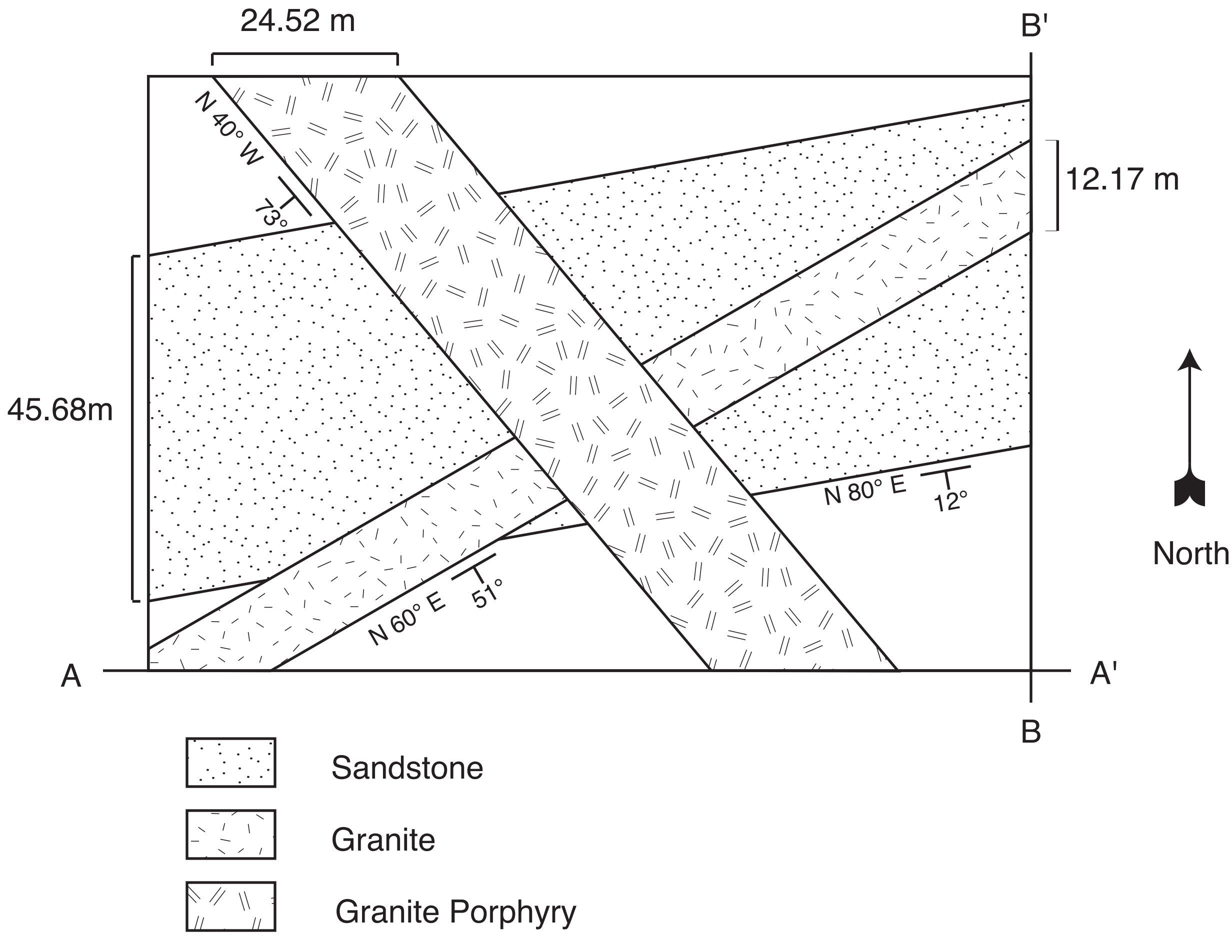
3. What is the thickness of the igneous intrusion?

4. What is the thickness of the sandstone bed?

5. What is the horizontal extent of the igneous intrusion in the E-W direction?

6. What is the horizontal extent of the sandstone bed in the E-W direction?

Three Unit Cross Section Problem



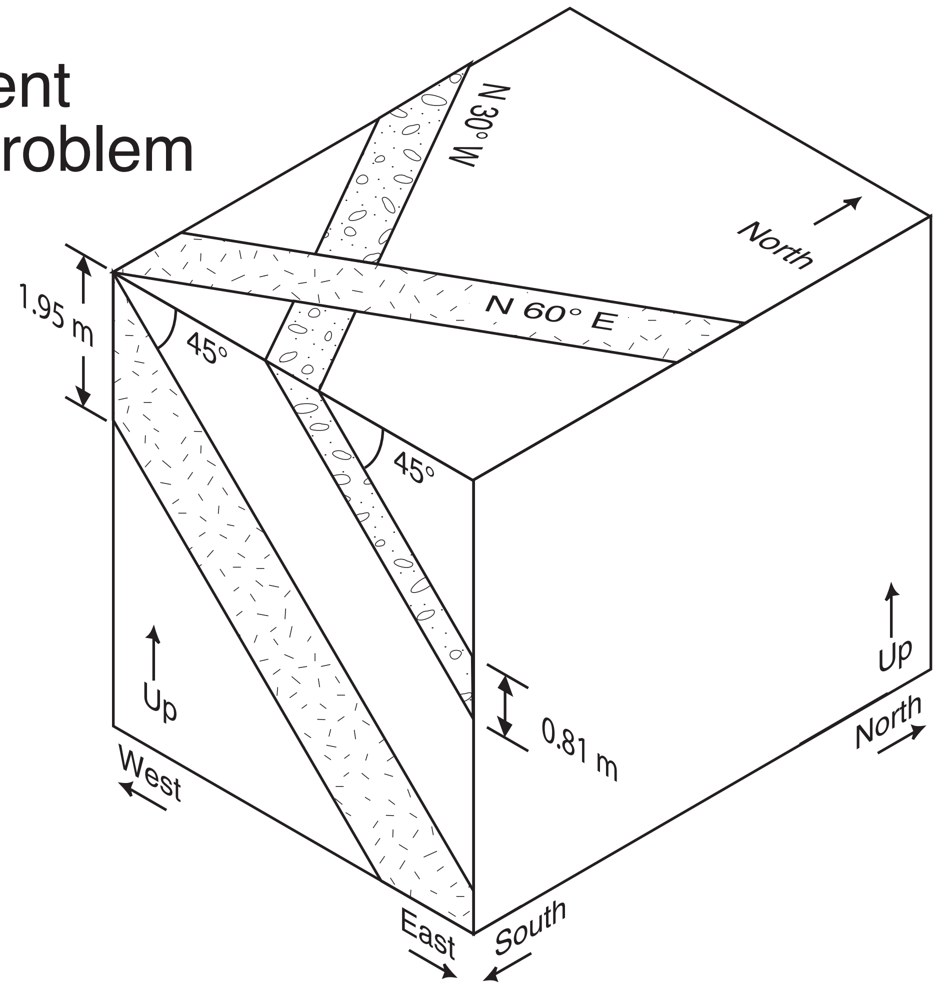
1. Construct vertical cross sections along A - A' and B - B'.

2. What is the thickness of the sandstone bed?

3. What is the thickness of the Granite dike?

4. What is the thickness of the Granite Porphyry dike?

Same Apparent Dips Problem



A bed of conglomerate striking N 30° W and a planar igneous intrusion striking N 60° E both have apparent dips of 45° to the East, as shown in the illustration above. The vertical extent of the conglomerate bed is 0.81 m, and the vertical extent of the igneous intrusion is 1.95 m. What is the:

a. True dip of the Conglomerate bed? ________________

b. True dip of the planar igneous intrusion? ________________

c. Apparent dip of the Conglomerate bed in the N-S plane? ________________

d. Apparent dip of the planar igneous intrusion in the N-S plane? ________________

e. Orientation of the intersection of the bed and intrusion? ________________

f. Angle between the traces of the bed and intrusion on the N-S plane? ________________

g. Thickness of the conglomerate bed? ________________

h. Thickness of the igneous intrusion? ________________

Playing With Thickness...

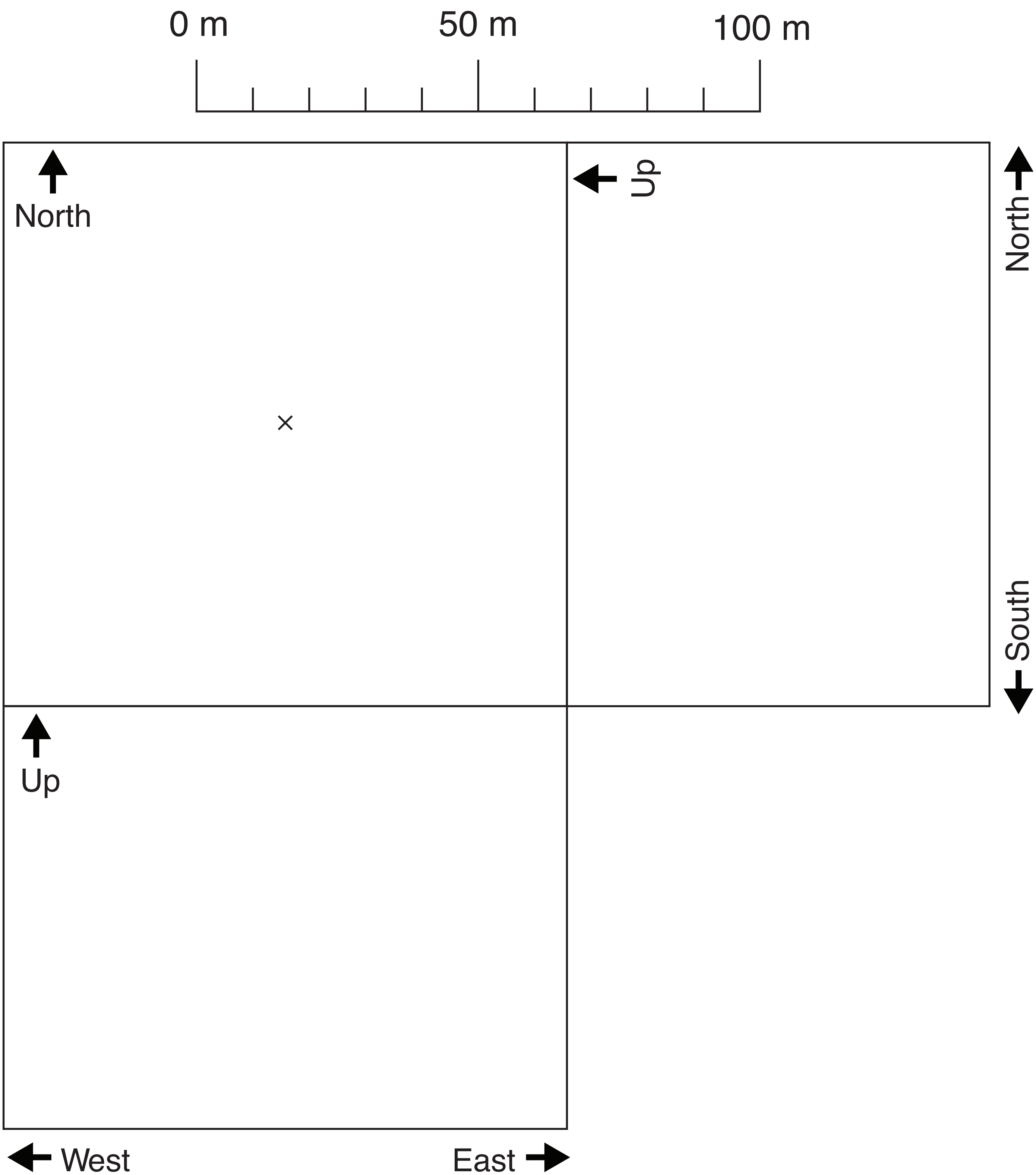
At the "X" shown in the map below there is an intersection between:

The upper surface of a 5.69 m thick sandstone bed, with an attitude of N 35 E, 10 E

and

The lower surface of a 16.28 m thick dike, with an attitude of N 20 W, 60 E.   (These are true thicknesses...)

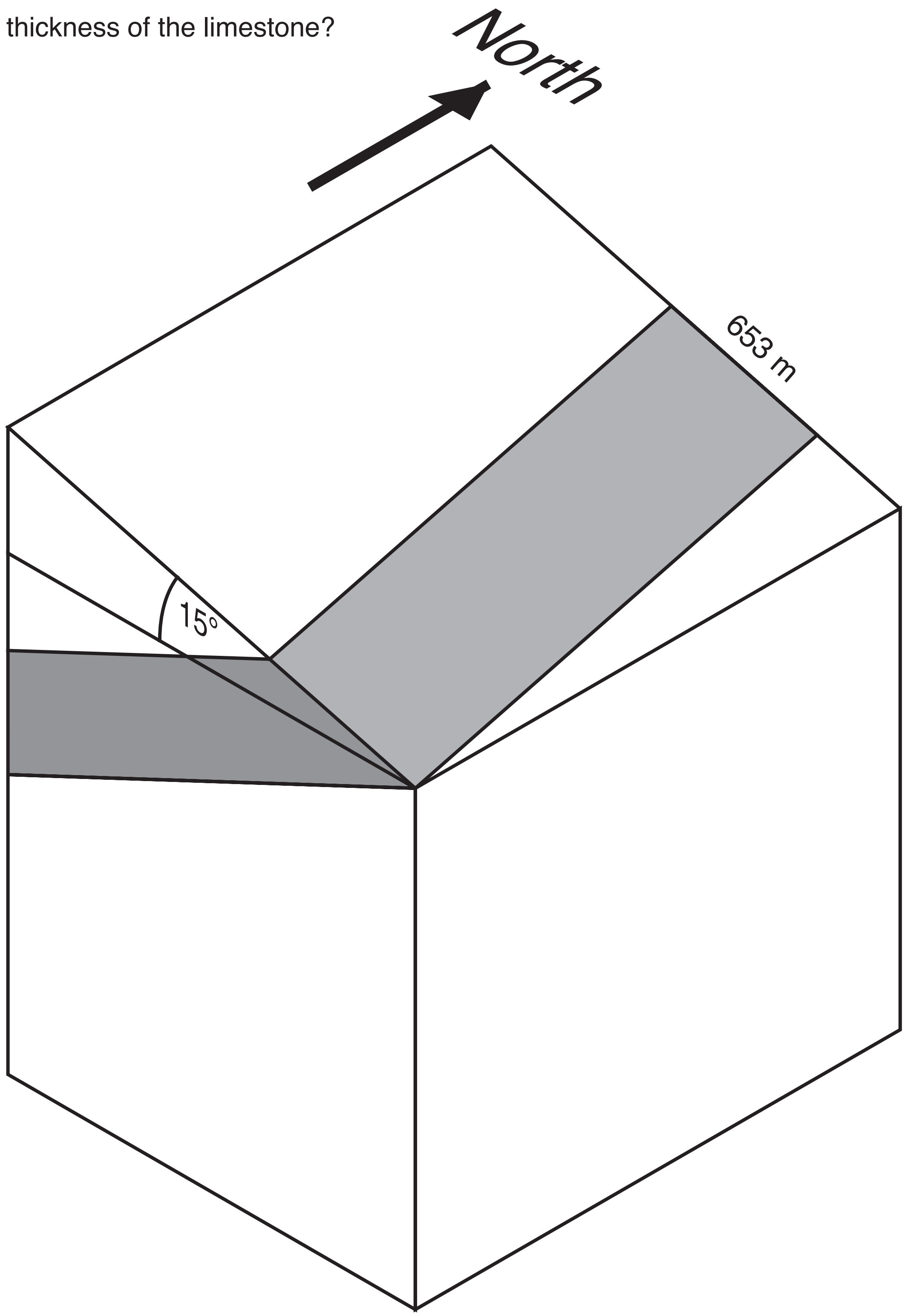
Complete the map and the two vertical cross sections using the scale indicated for both horizontal and vertical scale.



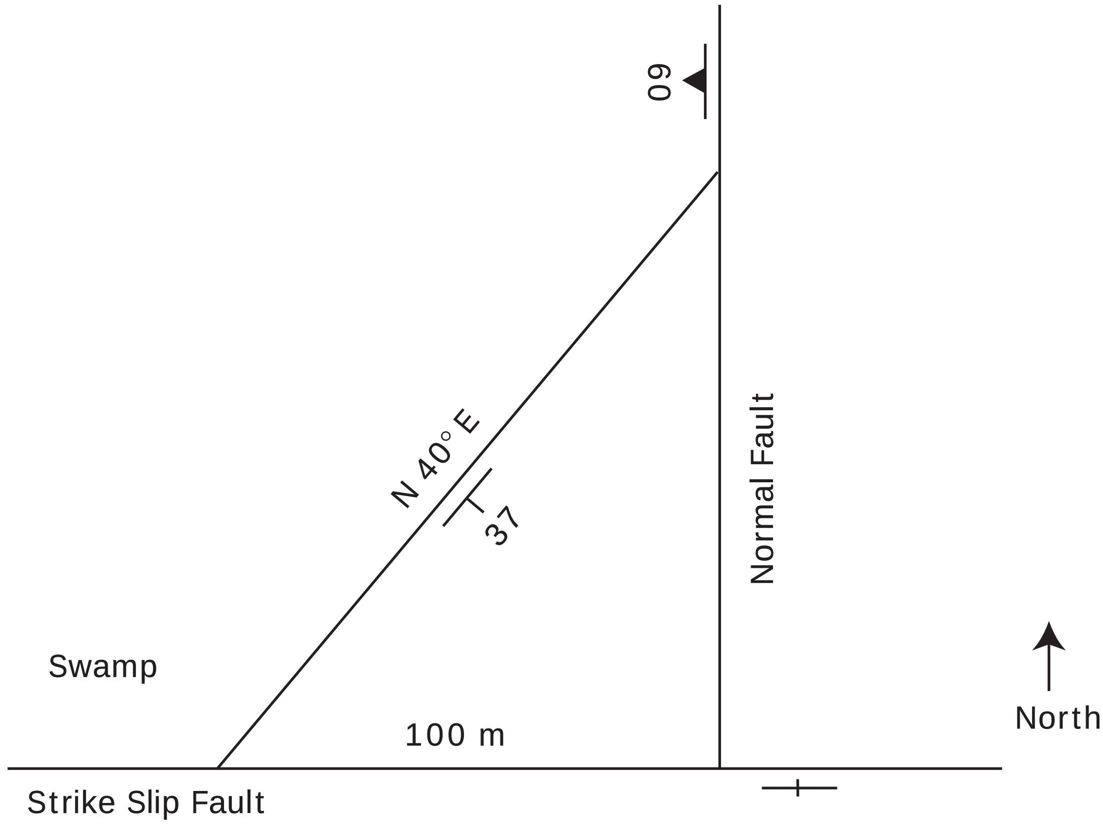
From Donal Ragan, Chapter 2:
Problem 2.

A limestone formation is exposed on an east facing slope. Its attitude is N 15° W, 26° W. The traverse length from the bottom made with a bearing of N 90° W is 653 m, and the slope angle was 15° upward.

What is the thickness of the limestone?



Structures Hydrology Problem



The town of Dedaichtuoh has a problem. It has been getting its water for years from a 63.88 m thick unit, the Mambo Sandstone, striking N40E, dipping 37 SE, which has a porosity of 20%. As it grew and developed, its need for water increased. New and deeper wells were drilled. Soon folks found that the unit beneath the Mambo Sandstone was an impermeable shale, and the subsurface extent of the Mambo Sandstone was limited by the juxtaposition of other impermeable units when it extended to either a N-S normal fault, with a dip of 60° W, or a vertical strike-slip fault extending E-W. Worse yet, whenever a well was drilled to a depth greater than 60 m, remaining in the Mambo Sandstone, it encountered Creosote. Discovering this Creosote should not have been a complete surprise, as a plant treating telephone poles with this DNAPL pollutant had been operating in the town until only a few years ago. Faced with a potentially large cleanup or mitigation bill, the town has hired you to estimate the amount of Creosote that they have to deal with.

When you go to help them out, you find that the top of the Mambo Sandstone crops out 100 m to the West of the intersection of the two faults, as shown in the sketch map. How much Creosote is lurking down there?

(The volume of an irregular tetrahedron with one face horizontal, is one third of the area of that face times the difference in elevation between that face and the other corner.)

Directions for 101 Faults

Strike and Dip of Bedding

Strike and Dip of Fault Surface

The figures on the attached sheet are in map view, with North towards the top of the page.

Assume no major horizontal motion on the faults (that is, they are either Normal or Reverse faults).

1 - 4: Label upthrown and downthrown sides of the fault. Determine whether the fault is normal or reverse.

5 - 12: Label upthrown and downthrown sides of the fault. Determine whether the fault is normal or reverse. Assuming that the area is simply folded (i.e. antiforms are anticlines, etc.), determine whether the fold is an anticline or a syncline. Show direction of plunge with an arrow.

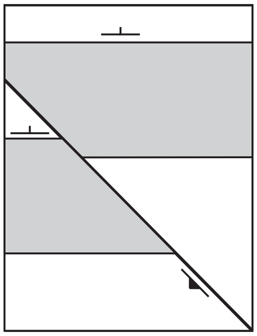
13. Draw a map showing an anticline and a syncline plunging to the East.

14. Draw a map showing an anticline plunging to the North which has been cut by a normal fault which runs N-S through the center of the fold and uplifts the East side.

15. Draw a map showing a syncline plunging to the South which has been cut by a reverse fault which runs N-S throught the center of the fold and uplifts the West side.

16. Draw a map showing a dome which has been cut by a fault which uplifted half of it. Label upthrown and downthrown sides, the dip of the fault surface and indicate whether normal or reverse.

1



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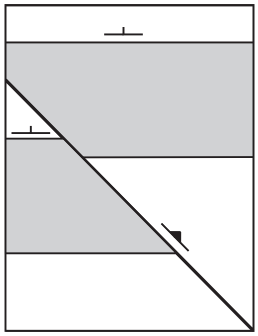

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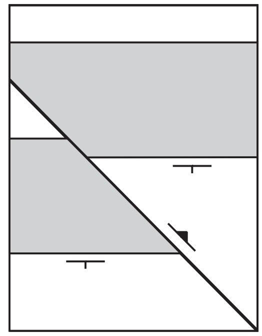

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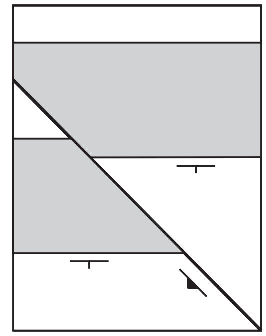

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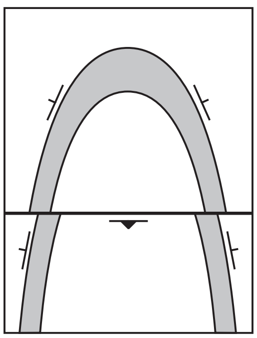

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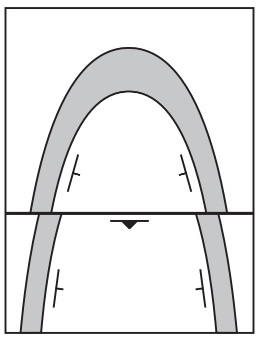

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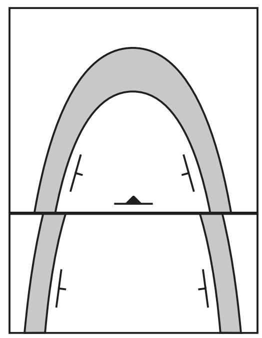

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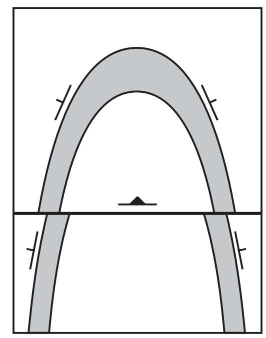

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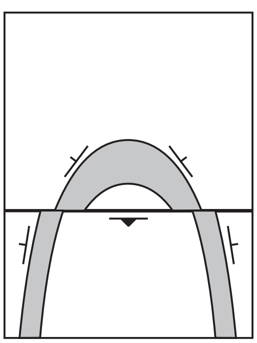

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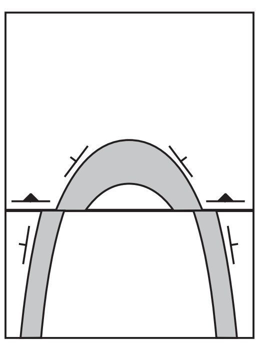

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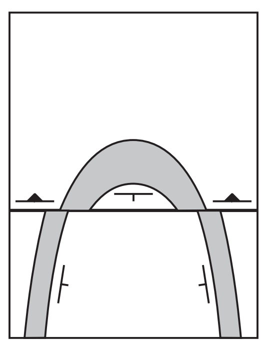

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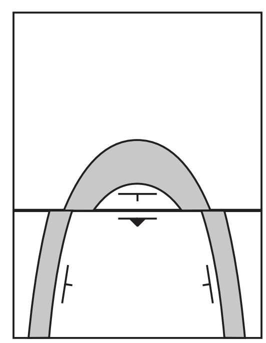

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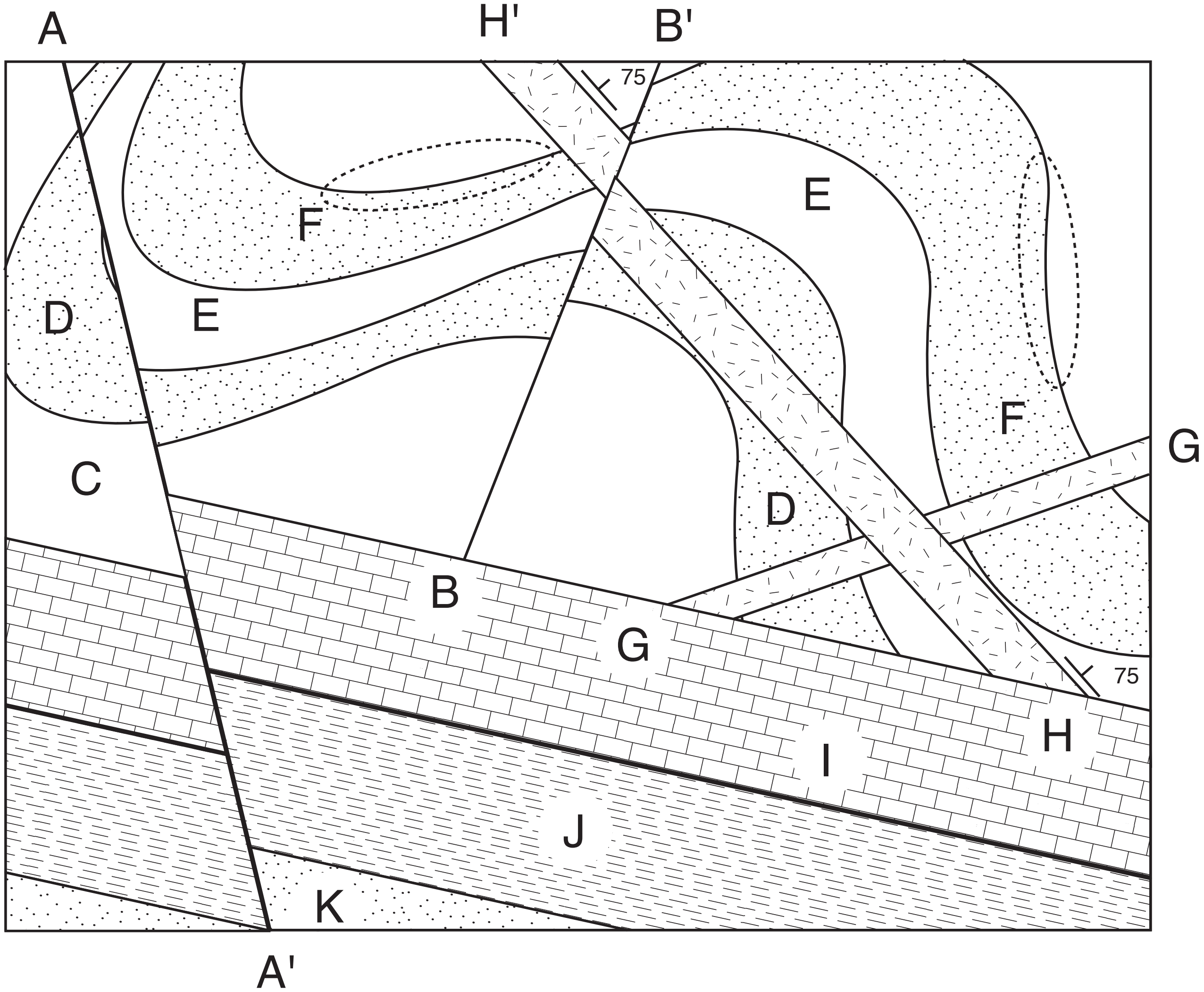
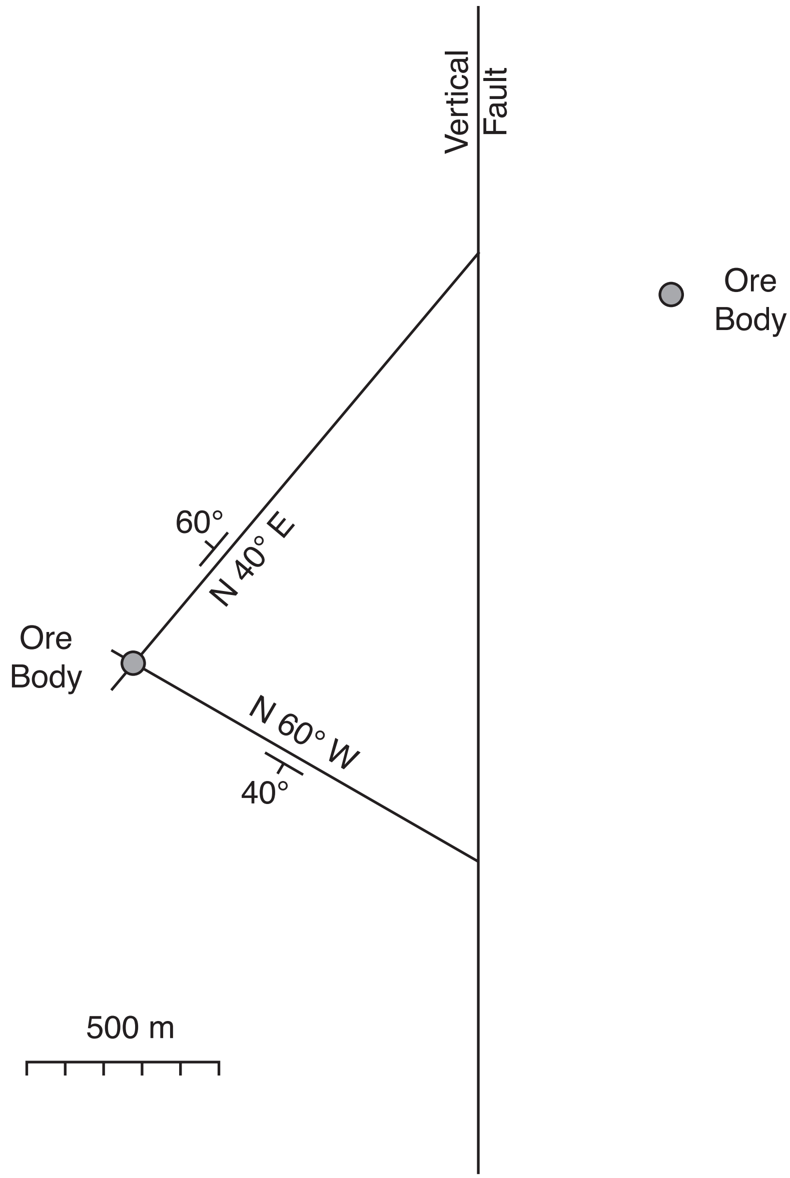


1. Put Strike and Dip symbols inside the dashed ellipses

2. Label Upthrown and Downthrown side of faults

3. Put an "S" in the nose of the synform(s) and an "A" in the nose of the antiform(s)

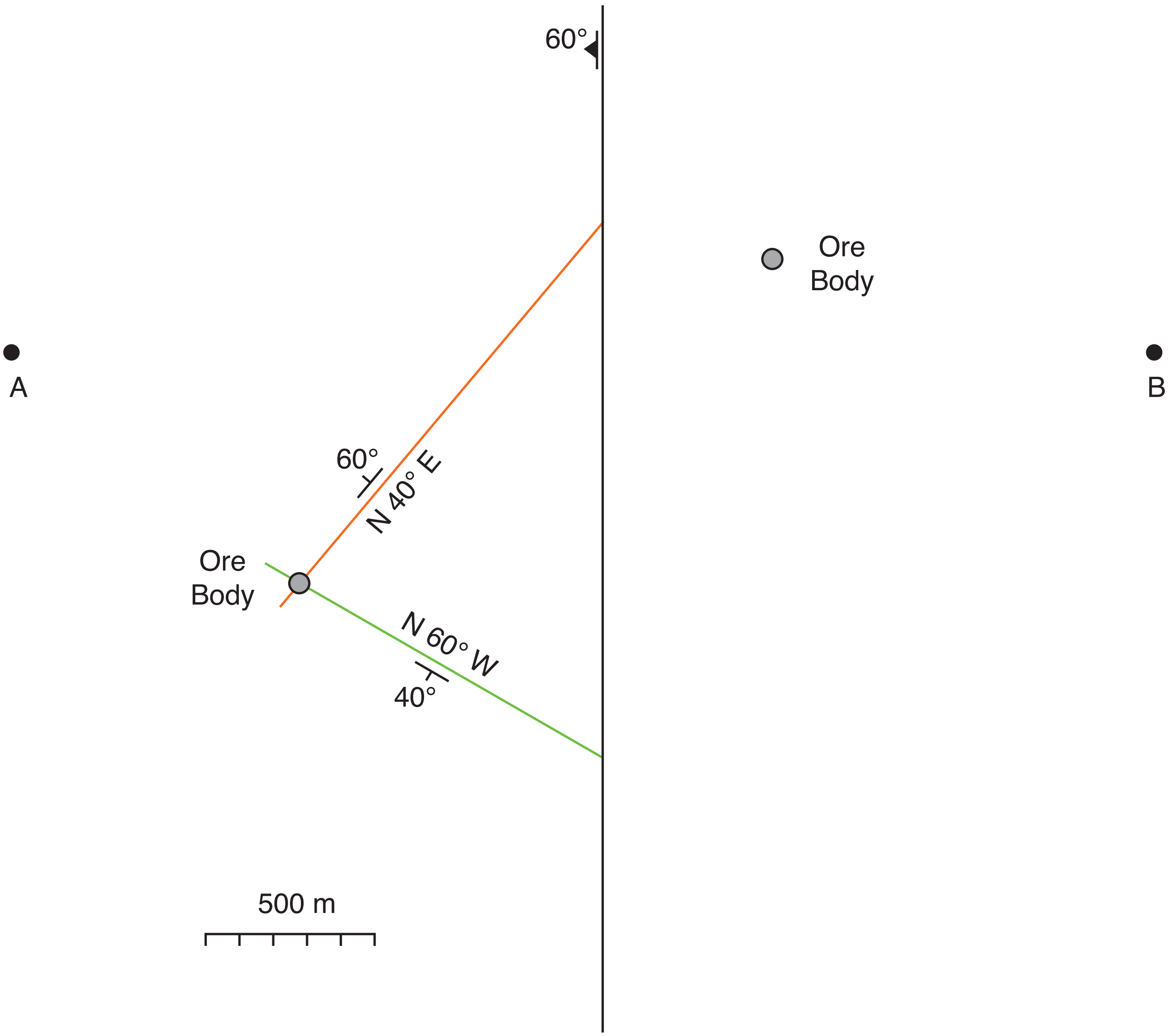
4. Carefully describe the geologic history of the mapped area, starting with the deposition of the oldest unit mapped. Be sure to include all erosion, uplift, subsidence, deformation and intrusion events at their proper place in the sequence.



An ore body is found along the intersection of a limestone bed and a shear zone. The limestone strikes N 60° W and dips 40° SW, the shear zone strikes N 40° E and dips 60° to the Northwest. East of a North-South vertical fault the ore body is found to be 1400 m East and 960 m North of its outcrop to the West of the fault. What is the net slip on the fault?

Is it necessary to know where the fault is?

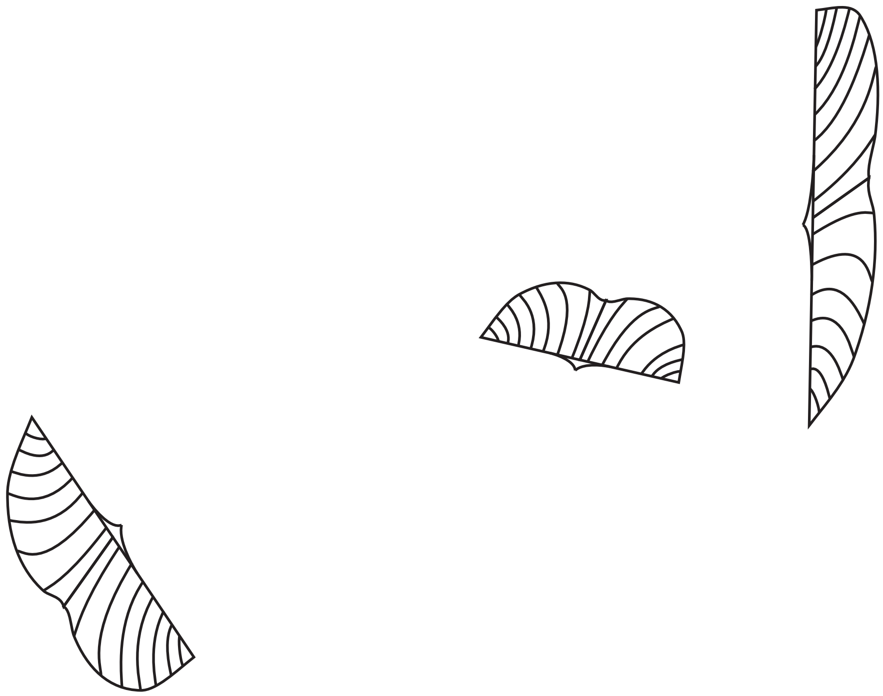
Another Ore Body Problem

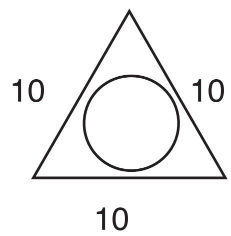


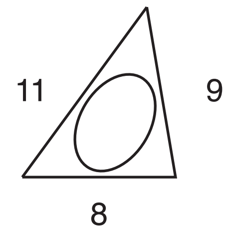
An ore body is found along the intersection of a limestone bed and a shear zone. The limestone strikes N 60° W and dips 40° SW, the shear zone strikes N 40° E and dips 60° to the Northwest. East of a North-South fault dipping 60° W the ore body is found to be 1400 m East and 960 m North of its outcrop to the West of the fault. What is the net slip on the fault?

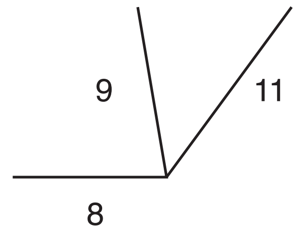
Construct a vertical cross section from A to B.

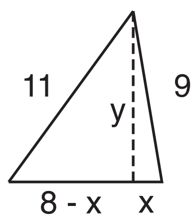
Use Ramsay’s Method (Marshak and Mitra, page 348) to determine the strain experienced by the three brachiopods shown.





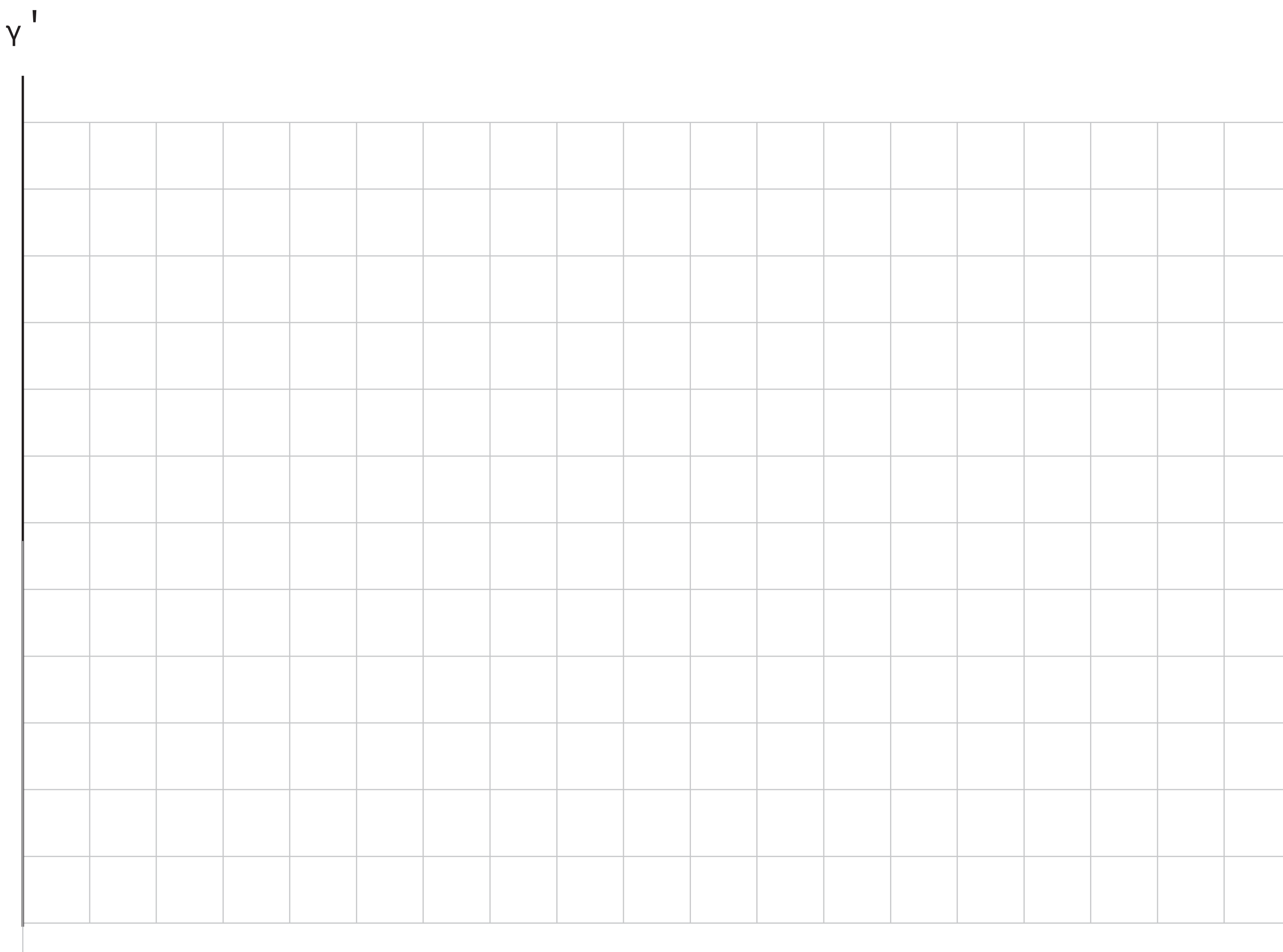




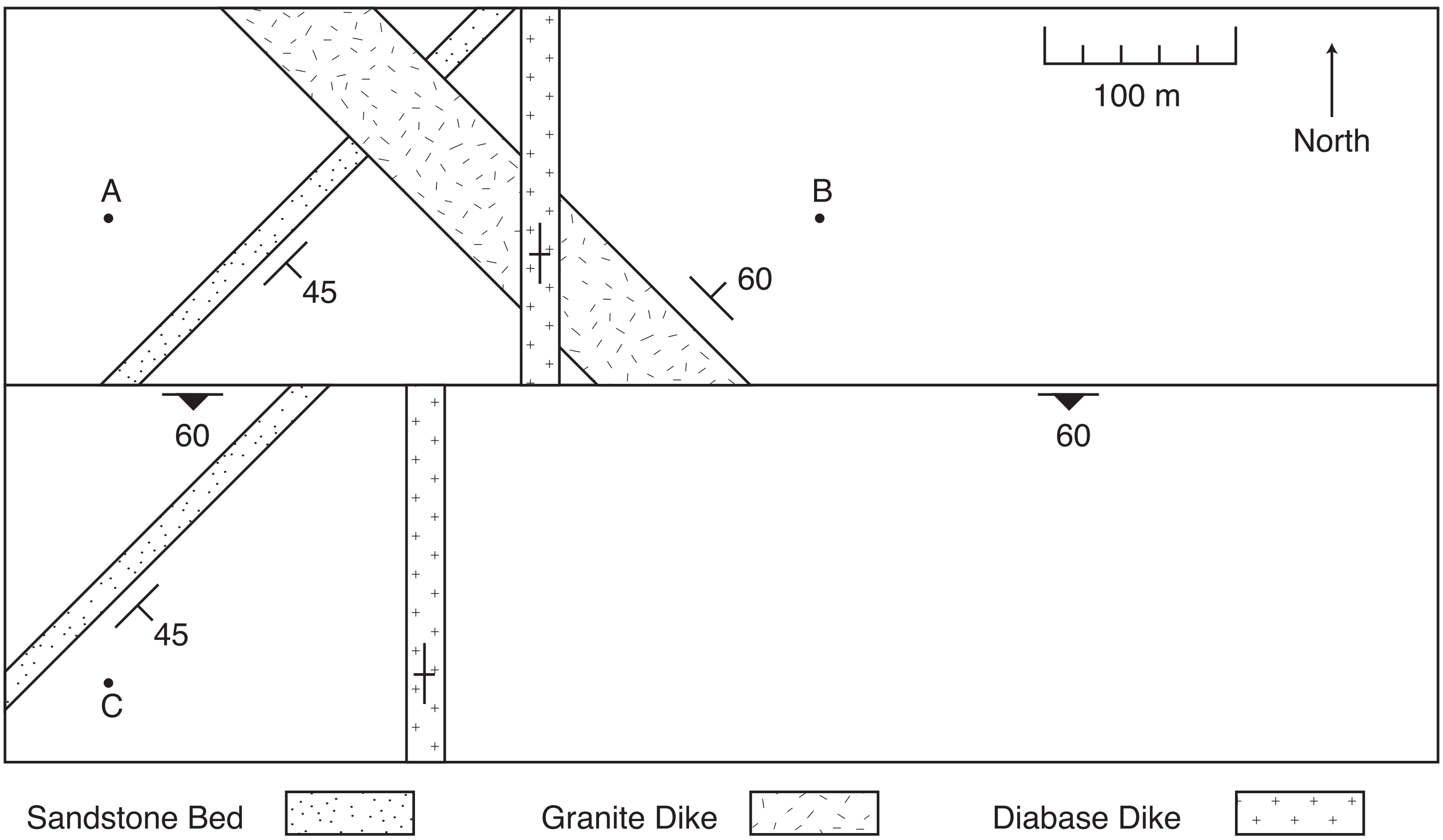


$y^2 + (8 - x)^2 = 121$
$y^2 + x^2 = 81$
$64 - 16x = 40$
$x = 1.5$

Left lower angle = arccos (6.5/11) = 53.7784°
Righ lower angle = arccos(1.5/9) = 80.4059°
Top angle = 180 - 53.7784 - 80.4059 = 45.8157°



Fault Displacement Problem



A sandstone bed has an attitude of N 45 E, 45 SE. It is cut by a granite dike with an attitude of N 45 W, 60 NE, which is cut in turn by a vertical diabase dike trending N - S. All of these are disrupted further by a fault striking E - W and dipping 60 to the South.

1. What is the strike slip component of the net displacement?

2. What is the dip slip component of the net displacement?

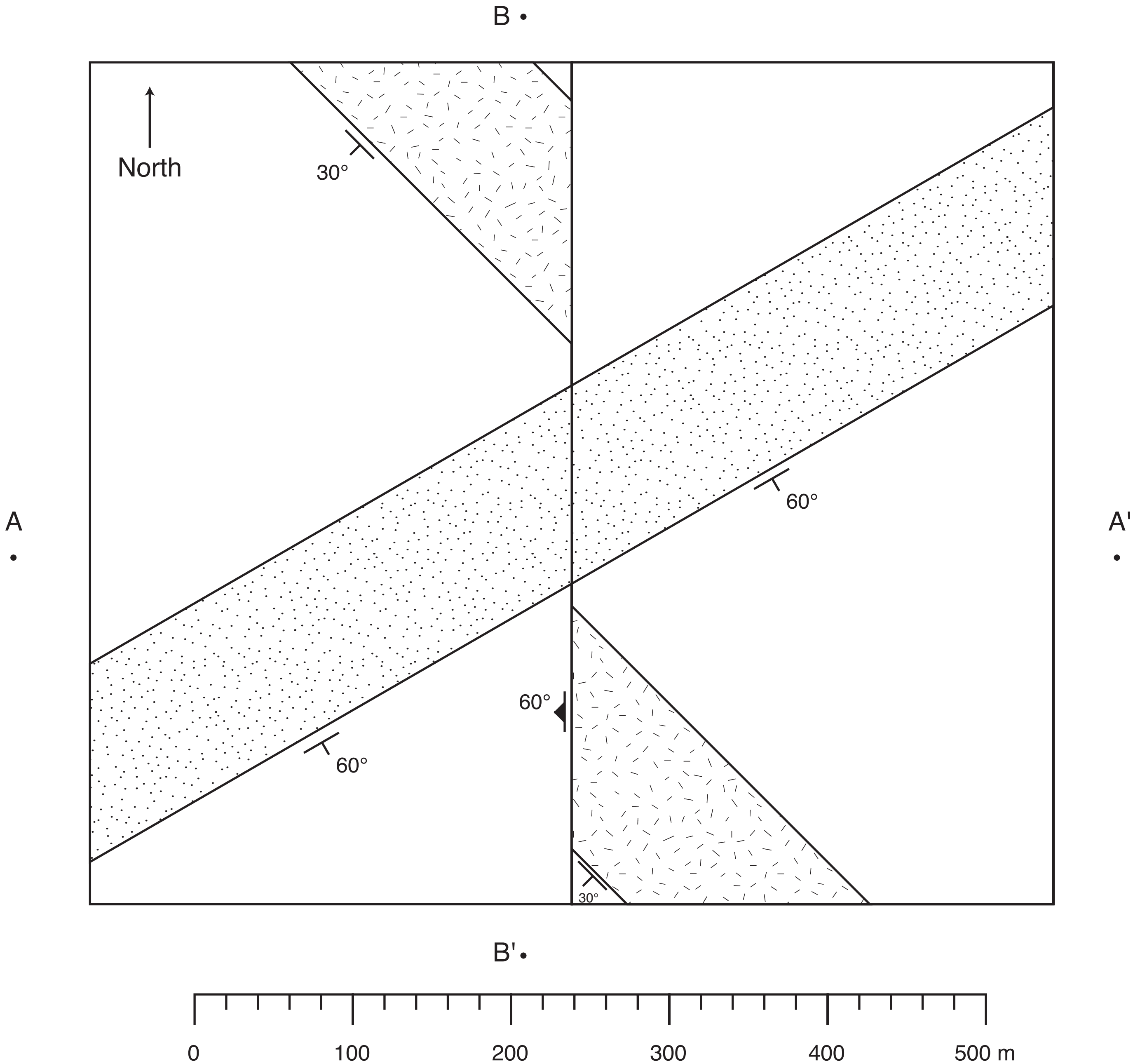
3. What is the magnitude, plunge and bearing of the net displacement?

4. Complete the map by drawing in the granite dike south of the fault.

5. Construct a vertical cross section from A to B.

6. Construct a vertical cross section from A to C.

Structural Geology Fault Homework



Sandstone bed strikes N 60° E, dips 60° SE.

Granite dike strikes N 45° W, dips 30° SW.

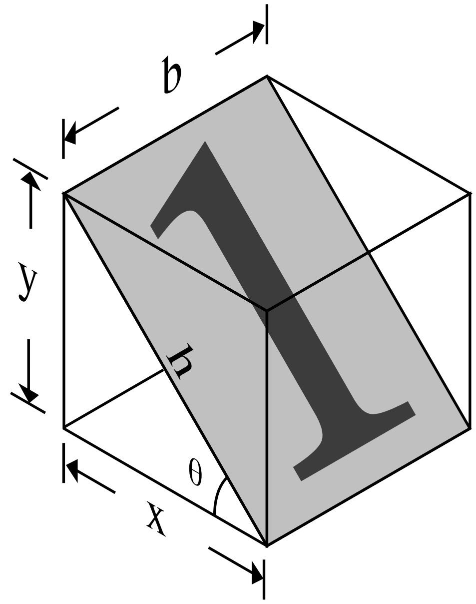
Fault strikes N-S and dips 60° to the West.

1. What is the net slip on the fault? ______________ What kind of fault is it? _____________

2. How thick is the sandstone bed? ____________ The granite dike? _______________

3. Draw vertical cross sections showing structure above and beneath A - A' and B - B'.

We wish to determine the stresses acting on a plane with an arbitrary orientation. Let the angle between this plane and the plane normal to the maximum stress (σ_1) be given as θ.



Next, we note that as we are working with a two dimensional stress system, we can adjust the size of the third dimension (b) such that the area of the plane in question is equal to one.

This means that b * h = 1 or that b = 1 / h .

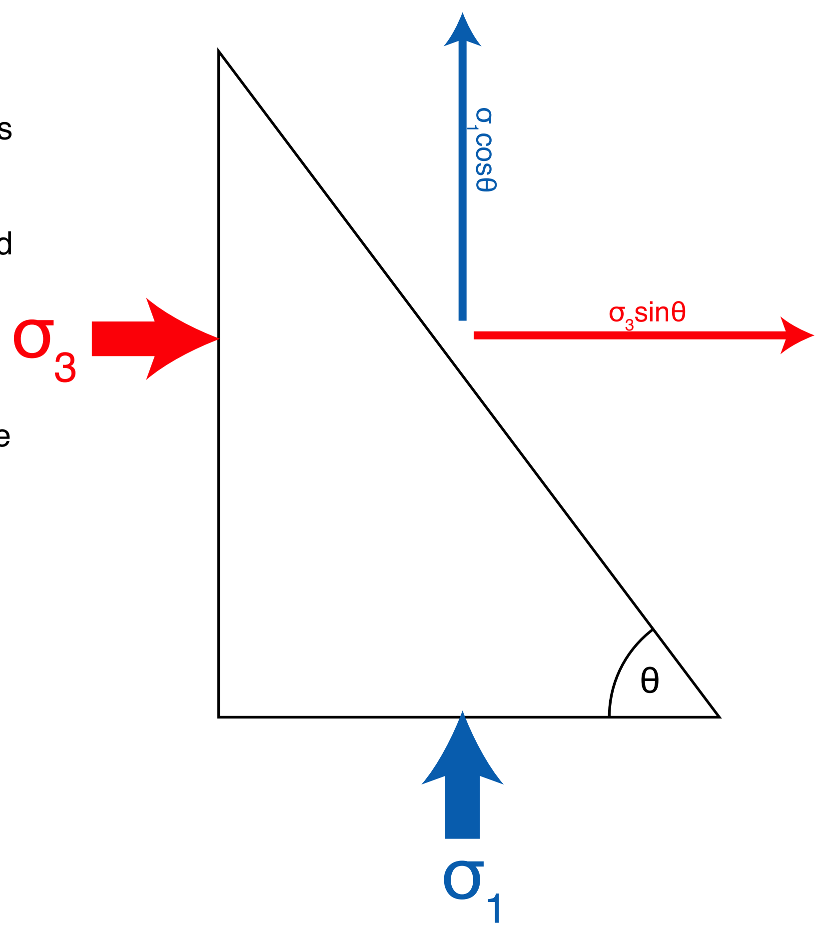
Because cos θ = x / h, x = h cos θ, and so the bottom of the box, the area over which σ_1 is acting, is equal to x * b, or cos θ.

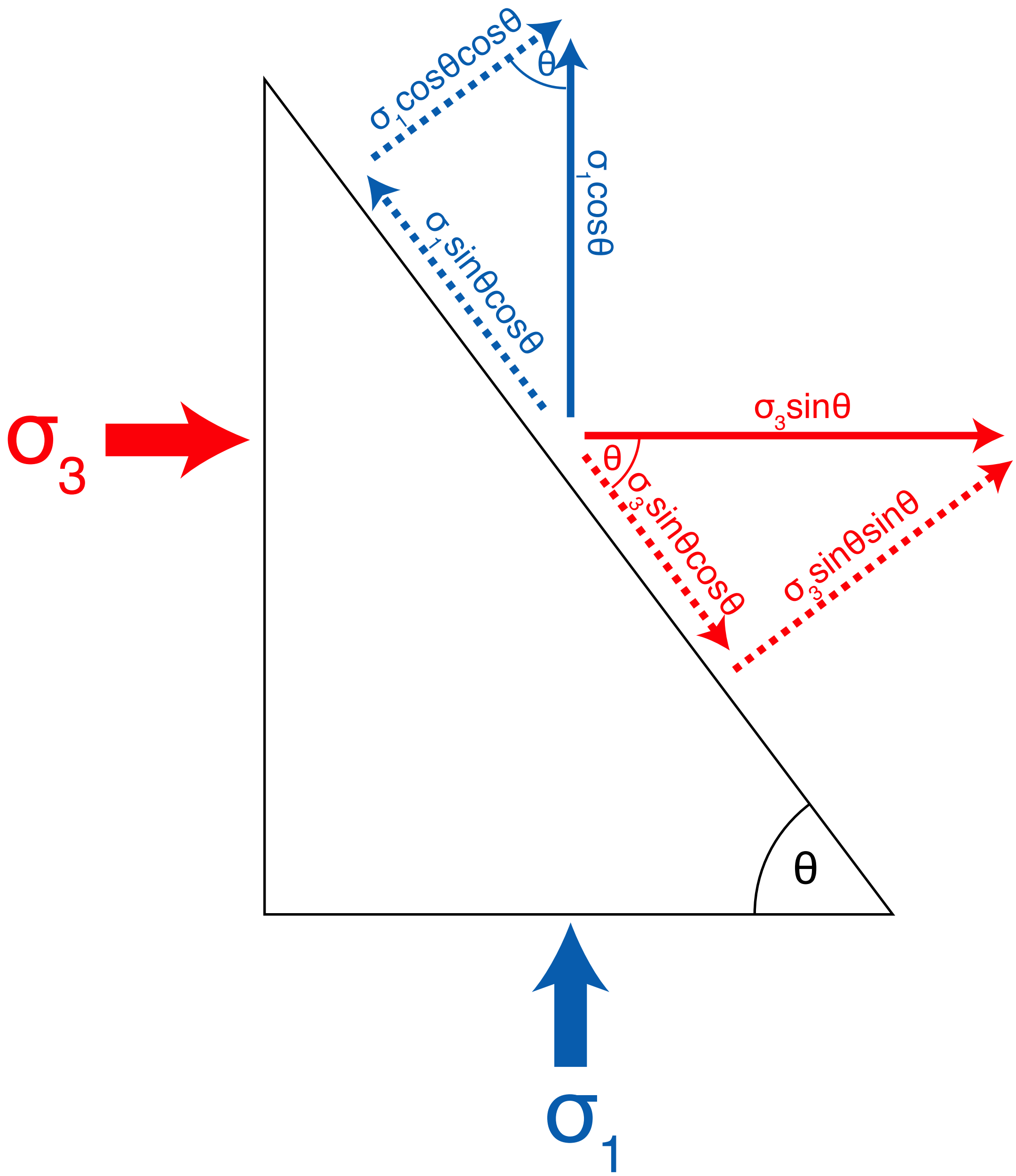
Because sin θ = y / h, y = h sin θ, and so the side of the box, the area over which σ_3 is acting, is equal to y * b, or sin θ.

Knowing these areas, we can get the forces acting on our plane by multiplying each stress times the area over which it is acting.

We could, if we wished, now add these forces together to get the net force acting on our plane. Note that we could not add, or resolve, the stresses. We needed to convert them to forces, by multiplying times the areas over which they acted.

Forces are vectors, and so can be resolved into components in various directions. For our purposes, it is useful to resolve them into those components which are normal (perpendicular) to the plane in question, called “normal forces,” and those which are parallel to the plane in question, which are called “shear forces.”



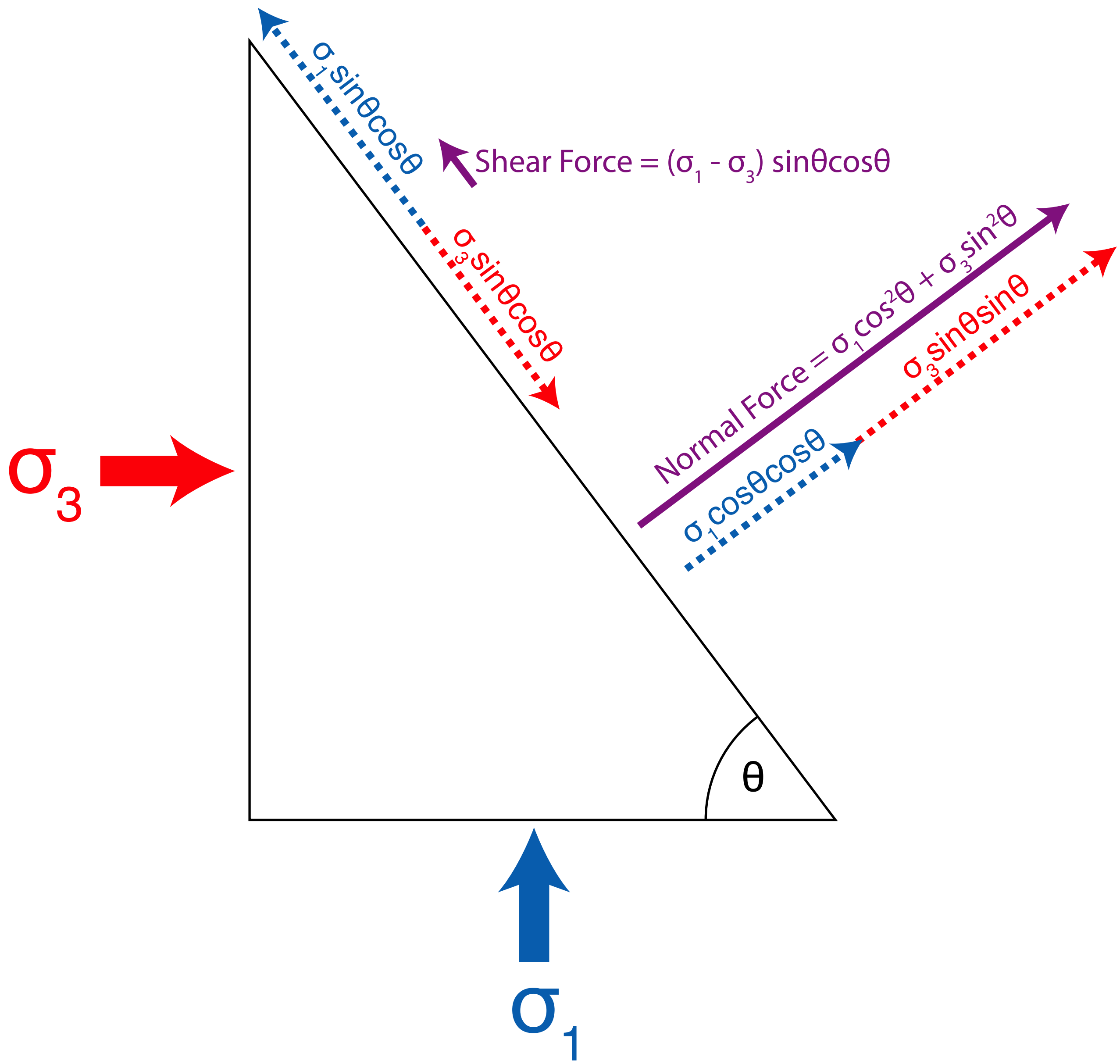


First, we need to establish our geometrical relationships:

The red θ is equal to the black θ because of "parallel lines cut by a straight line" and the blue θ must equal the black θ because the other angles in the blue triangle are 90° and (180° - 90° -θ).

Then we can resolve each of the solid colored vectors into two components, one normal to, and the other parallel to, the plane in question.

These components can then be added, recalling that to add vectors you put the tail of one on the head of the other.



Now that we resolved our forces into components, added the components and determined the resultant forces acting normal and parallel to our plane, we can convert these forces back into stresses by dividing by the area over which they act. But, aha! We started by stipulating that this area would be equal to one... (weren't we clever?).

Shear stress $\sigma_\tau = (\sigma_1 - \sigma_3) \sin\theta \cos\theta$ Or, using earlier results: $\sigma_\tau = [(\sigma_1 - \sigma_3) / 2] \sin 2\theta$

Normal stress $\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$

But, from earlier, we know that $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

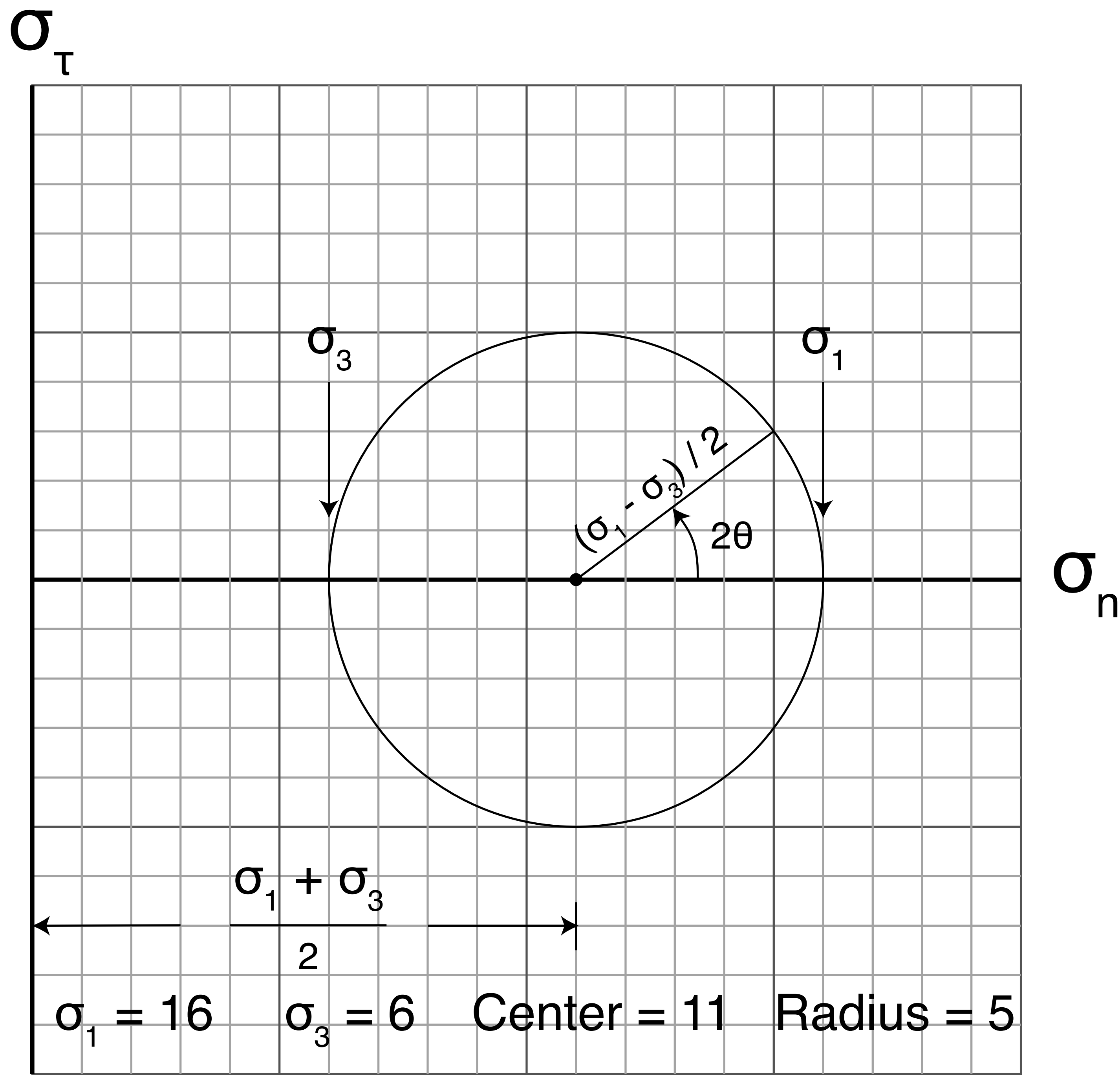
Rearranging: $\cos^2\theta = (1 + \cos 2\theta) / 2$ and $\sin^2\theta = (1 - \cos 2\theta) / 2$

Giving us $\sigma_n = [\sigma_1 (1 + \cos 2\theta) / 2] + [\sigma_3 (1 - \cos 2\theta)] = (\sigma_1 + \sigma_3) / 2 + [(\sigma_1 - \sigma_3) / 2] \cos 2\theta$

As can be shown, these are the parametric equations for a circle, in terms of 2θ, with a radius of $(\sigma_1 - \sigma_3) / 2$ and a center at $(\sigma_1 + \sigma_3) / 2$.

Recall that the angle θ is the angle between the plane in question and the plane which is normal to the σ_1 direction. It is zero when the plane is being acted on by only σ_1, and will be 90° (that is, 2θ will be 180°) when the plane is being acted on by only σ_3.

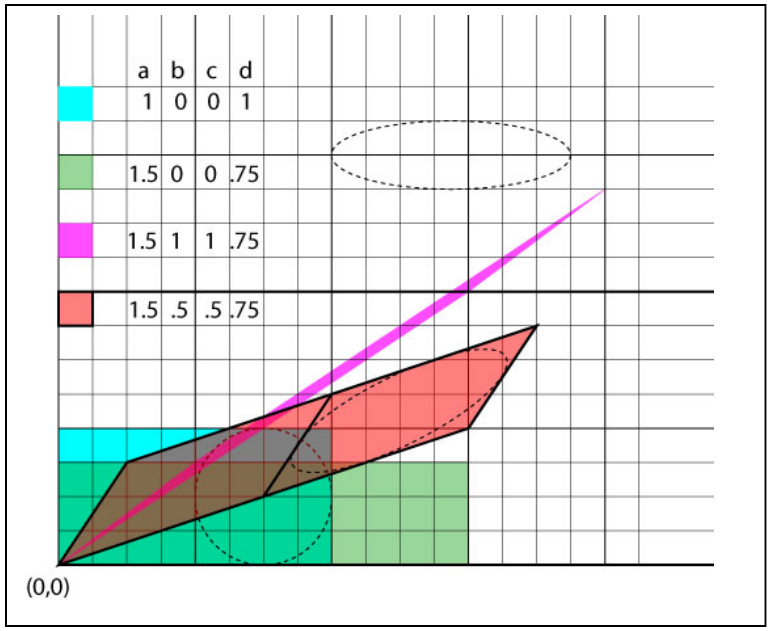
This is illustrated below:



Consider:

$x_1 = ax + by \quad y_1 = cx + dy \quad a \neq 0,\ d \neq 0$

Examples on the right show this for an original body in blue, and three strained bodies in green and purple, and red.



This represents the General Linear Transformation. We first wish to make sure that a deformation of this type gives Homogeneous Strain, where straight, parallel lines remain straight and parallel. We note that we can solve the equations above to get x and y in terms of x_1 and y_1:

$$
\begin{array}{cc}
cx_1 = cax + cby & dx_1 = dax + dby \\
\underline{-(ay_1 = acx + ady)} & \underline{-(by_1 = bcx + bdy)} \\
cx_1 - ay_1 = (cb - ad)y & dx_1 - by_1 = (ad - bc)x \\
\text{so} & \text{so} \\
y = \dfrac{cx_1 - ay_1}{cb - ad} & x = \dfrac{dx_1 - by_1}{ad - bc}
\end{array}
$$

Now, suppose we begin with a straight line in the undeformed space having an equation y = mx + k. According to our transformation, this line will have an equation in the deformed state of

$$\frac{cx_1 - ay_1}{cb - ad} = \frac{mdx_1 - mby_1}{ad - bc} + k$$

multiply the left hand side by $\frac{-1}{-1}$

$$\frac{ay_1 - cx_1}{ad - bc} = \frac{mdx_1 - mby_1}{ad - bc} + k$$

Multiply both sides by the denominator:

$$ay_1 - cx_1 = mdx_1 - mby_1 + k(ad - bc)$$

$$y_1 = \left(\frac{c + md}{a + mb}\right)x_1 + k\frac{ad - bc}{a + mb}$$

This is the equation for a straight line, so this transformation results in a straight line, and all lines with an undeformed slope of "m" will have a slope in the deformed state of $\left(\frac{c + md}{a + mb}\right)$ and so will be parallel.

How will the unit circle, $x^2 + y^2 = 1$, transform by this general linear transformation? Substituting our results for x and y we find:

$$c^2 x_1^2 - 2cax_1y_1 + a^2y_1^2 + d^2x_1^2 - 2bdx_1y_1 + b^2y_1^2 = a^2d^2 - 2abcd + b^2c^2$$

or

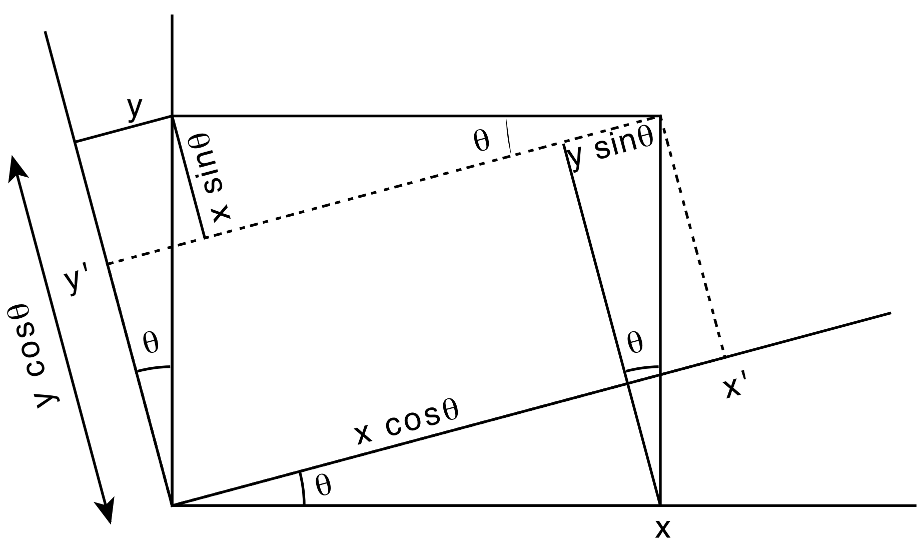
$$(c^2 + d^2)x_1^2 - 2(ac+bd)x_1y_1 + (a^2 + b^2)y_1^2 = (ad - bc)^2$$

This equation looks messy, and we may "simplify" it by defining some new constants in terms of our old constants... Let

$$\frac{c^2 + d^2}{(ad - bc)^2} = \mathbf{l} \qquad \frac{ac + bd}{(ad - bc)^2} = \mathbf{m} \qquad \frac{a^2 + b^2}{(ad - bc)^2} = \mathbf{n}$$

Now our equation looks like: $\boldsymbol{l}\, x_1^2 - 2\boldsymbol{m}\, xy + \boldsymbol{n}\, y_1^2 = 1$

This may be "simplified" further, if we introduce new axes which have undergone rotation:



$$x' = x\cos\theta + y\sin\theta$$

$$y' = y\cos\theta - x\sin\theta$$

Then:

$$(x')^2 = x^2\cos^2\theta + 2xy\sin\theta\cos\theta + y^2\sin^2\theta$$

$$(y')^2 = y^2\cos^2\theta - 2xy\sin\theta\cos\theta + x^2\sin^2\theta$$

$$(x'y') = xy\cos^2\theta + y^2\sin\theta\cos\theta - x^2\sin\theta\cos\theta - xy\sin^2\theta$$

big deal....

If we look at our equation for the deformed circle, and assume that it is in the rotated coordinated system, we can find what rotation, θ, is needed to get rid of the coefficient of the xy term. So our equation becomes

$$l(x')^2 - 2m(x'y') + n(y')^2 = 1$$

or $lx^2\cos^2\theta + \mathbf{2lxy\sin\theta\cos\theta} + ly^2\sin^2\theta - \mathbf{2mxy\cos^2\theta} - 2my^2\sin\theta\cos\theta + 2mx^2\sin\theta\cos\theta +$
$\mathbf{2mxy\sin^2\theta} + ny^2\cos^2\theta - \mathbf{2nxy\sin\theta\cos\theta} + nx^2\sin^2\theta = 1$

Collecting the terms in bold (those with xy in them), we can set it equal to zero so that it will vanish for all x and y:

$$2lxy\sin\theta\cos\theta - 2mxy\cos^2\theta + 2mxy\sin^2\theta - 2nxy\sin\theta\cos\theta = 0$$

Divide through by 2xy to get $\quad l\sin\theta\cos\theta - m\cos^2\theta + m\sin^2\theta - n\sin\theta\cos\theta = 0$

Or $$(l - n)\sin\theta\cos\theta - m(\cos^2\theta - \sin^2\theta) = 0$$

$$\frac{l-n}{m} = \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\frac{\sin 2\theta}{2}} = \frac{2}{\tan 2\theta}$$

Going back to where we defined l, m and n, we see that for the coefficient of the xy term to vanish:

$$\frac{c^2 + d^2 - a^2 - b^2}{ac + bd} = \frac{2}{\tan 2\theta}$$

$$\tan 2\theta = \frac{2\cdot(ac+bd)}{c^2 + d^2 - a^2 - b^2}$$

Finally: $$\theta = \frac{1}{2}\arctan\frac{2\cdot(ac+bd)}{c^2 + d^2 - a^2 - b^2}$$

For our purple body,

θ = 0.5 arctan { 2 * (1.5 + .75)} / (1 + 0.5625 - 2.25 - 1) = -34.722°
arctan(11/16) = 34.51° so this looks good

For our red body,

θ = 0.5 arctan { 2 * (.75 + .375)} / (.25 + 0.5625 - 2.25 - .25) = -26.565°

$$\frac{c^2 + d^2}{(ad - bc)^2} = \mathbf{l} \qquad \frac{ac + bd}{(ad - bc)^2} = \mathbf{m} \qquad \frac{a^2 + b^2}{(ad - bc)^2} = \mathbf{n}$$

denominator = $(1.125 - .25)^2 = .765625$

l = 1.0612   m = 1.46938   n = 3.2653 $\sin^2\theta = 0.2$ $\cos^2\theta = 0.8$ $\sin\theta\cos\theta = -0.4$

We had: $lx^2\cos^2\theta + ly^2\sin^2\theta - 2my^2\sin\theta\cos\theta + 2mx^2\sin\theta\cos\theta + ny^2\cos^2\theta + nx^2\sin^2\theta = 1$

which becomes

$$(.8 * 1.0612 + 2 *(- .4) * 1.469 + .2 * 3.265)x^2 + (.2 * 1.0612 - 2 * (- .4)* 1.469 + .8 * 3.265)y^2 = 1$$

$$0.3265\, x^2 + 4\, y^2 = 1$$

$$(x/1.75)^2 + (y/.5)^2 = 1$$

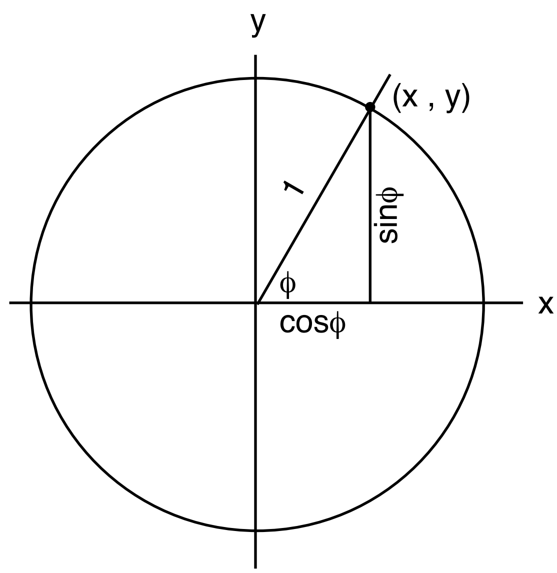
These results are for a unit circle. The circle drawn on the original blue rectangle on the first page had a diameter of 4, however.

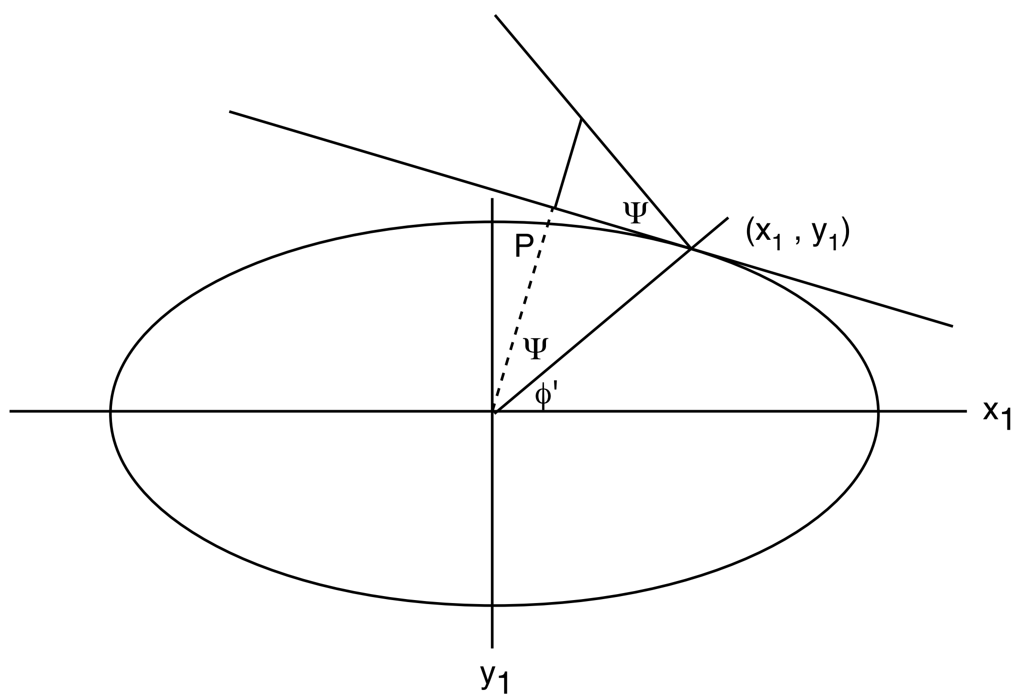
So the final ellipse will have a major axis of 7 and a minor axis of 2, and will be rotated by 25.565°.

This ellipse is drawn on the original diagram, in its rotated and unrotated orientations.

We have seen that for any arbitrary homogeneous (i.e. linear) deformation of a unit circle, the result will be an ellipse. The major and minor axes of this ellipse will not, in general, coincide with the coordinate axes, however the coordinate axes may be rotated to make them coincide. Hence any linear (homogeneous) deformation may be thought of as an irrotational strain, where the directions of greatest and least extension are parallel to the coordinate axes, followed by a rigid body rotation. Because it is usually not possible to detect or measure rigid body rotations in geologically relevant problems, we will now concentrate on irrotational strain.

Consider the deformation of the unit circle shown below:





How does the length of the line, originally oriented at an angle ϕ from the x axis, change as the unit circle is deformed? Originally that line extends from the origin to point (x,y) where $x = \cos\phi$ and $y=\sin\phi$, so its length equals $\sqrt{(\cos^2\phi + \sin^2\phi)} = 1$. (This is good, because that is what the radius of a unit circle should be...) After deformation it extends from the origin to the point (x_1,y_1), where

$$x_1 = x\sqrt{\lambda_1} = \cos(\phi)\sqrt{\lambda_1}$$

$$y_1 = y\sqrt{\lambda_2} = \sin(\phi)\sqrt{\lambda_2}$$

so the length equals $\sqrt{(\lambda_1\cos^2\phi + \lambda_2\sin^2\phi)}$ which we will now call $\sqrt{\lambda}$. It would be more useful to know how much change there has been in the length of a line oriented with respect to the major axis of the strained ellipse (i.e. in terms of ϕ' rather than ϕ). We see that

$$\cos\phi' = \frac{x_1}{\sqrt{\lambda}} \text{ so } x_1 = \cos\phi'\sqrt{\lambda} \text{ and } \sin\phi' = \frac{y_1}{\sqrt{\lambda}} \text{ so } y_1 = \sin\phi'\sqrt{\lambda}$$

And going back to the equations just above, we see that we can substitute to obtain:

$$\cos\phi = \frac{x_1}{\sqrt{\lambda_1}} = \frac{\sqrt{\lambda}\cos\phi'}{\sqrt{\lambda_1}}$$

$$\sin\phi = \frac{y_1}{\sqrt{\lambda_2}} = \frac{\sqrt{\lambda}\sin\phi'}{\sqrt{\lambda_2}}$$

Recalling that $\sin^2\phi + \cos^2\phi = 1$, we see that

$$\frac{\lambda\cos^2\phi'}{\lambda_1} + \frac{\lambda\sin^2\phi'}{\lambda_2} = 1$$

or

$$\frac{1}{\lambda_1}\cos^2\phi' + \frac{1}{\lambda_2}\sin^2\phi' = \frac{1}{\lambda}$$

and letting $\frac{1}{\lambda_1} = \lambda'_1$; $\frac{1}{\lambda_2} = \lambda'_2$; $\frac{1}{\lambda} = \lambda'$

$$\lambda' = \lambda'_1\cos^2\phi' + \lambda'_2\sin^2\phi'$$

What is the shear strain associated with the deformation of the line originally at an angle of ϕ with the x axis? Initially the tangent to the circle at point (x,y) made a right angle with this line. After deformation, the tangent to the ellipse at point (x_1',y_1') is no longer perpendicular to the line from the origin to point (x_1',y_1') but is different by an angle, ψ, the angle of shear. We wish to find ψ as a function of ϕ'. We begin by getting the equation for the deformed ellipse into a form where y_1 is a function of x_1 so we will be able to differentiate it to find its slope:

$$\frac{x_1^2}{\lambda_1}+\frac{y_1^2}{\lambda_2}=1 \quad \text{or} \quad y_1^2=\lambda_2\left(1-\frac{x_1^2}{\lambda_1}\right) \quad \text{or} \quad y_1=\sqrt{\lambda_2-\frac{\lambda_2 x_1^2}{\lambda_1}}$$

Now the slope of the tangent to the ellipse is given by the derivative:

$$\frac{dy_1}{dx_1}=\frac{1}{2}\left(\lambda_2-\frac{\lambda_2 x_1^2}{\lambda_1}\right)^{-\frac{1}{2}}\left(-2\frac{\lambda_2}{\lambda_1}x_1\right)=\frac{1}{2}\left(\frac{1}{y_1}\right)\left(-2\frac{\lambda_2}{\lambda_1}x_1\right) \quad \text{or} \quad \frac{dy_1}{dx_1}=-\frac{x_1\lambda_2}{y_1\lambda_1}$$

The tangent line will be a straight line with this slope passing through (x_1',y_1'). As it passes that point, we know its equation will be : $y_1=-\frac{x_1'\lambda_2}{y_1'\lambda_1}x_1+k$. Multiply both sides by $\frac{y_1'}{\lambda_2}$ to get

$$\frac{y_1' y_1}{\lambda_2}=-\frac{x_1' x_1}{\lambda_1}+k\frac{y_1'}{\lambda_2} \quad \text{or} \quad \frac{y_1' y_1}{\lambda_2}+\frac{x_1' x_1}{\lambda_1}=k\frac{y_1'}{\lambda_2}$$

Since (x_1',y_1') must be on the ellipse, this equation requires that $k\frac{y_1'}{\lambda_2}=1$ or $k=\frac{\lambda_2}{y_1'}$.

Recall that $x_1' = (\cos\phi)\sqrt{\lambda_1}$ and that $y_1' = (\sin\phi)\sqrt{\lambda_2}$, and this last equation becomes

$$\frac{x_1\cos\phi}{\sqrt{\lambda_1}}+\frac{y_1\sin\phi}{\sqrt{\lambda_2}}=1$$ as the equation of the tangent line.

Now, let's say we know the equation of a line is y = -mx + b. Then the line perpendicular to this will have a slope of $\frac{1}{m}$ and if it goes through the origin it will have the equation $y=\frac{x}{m}$. These two lines will intersect where $\frac{x}{m}=-mx+b$ or $b=\left(m+\frac{1}{m}\right)x=\frac{m^2+1}{m}x$.

So $x=\frac{bm}{m^2+1}$ hence $y=\frac{x}{m}=\frac{b}{m^2+1}$.

The distance, P, from the origin to this point is

$$P = \sqrt{x^2 + y^2} = \sqrt{\frac{b^2m^2}{\left(m^2+1\right)^2} + \frac{b^2}{\left(m^2+1\right)^2}} = b\sqrt{\frac{m^2+1}{\left(m^2+1\right)^2}} = \frac{b}{\sqrt{m^2+1}}$$

Substituting our values for b and m:

$$P = \frac{\dfrac{\lambda_2}{y'_1}}{\sqrt{\dfrac{\left(x'_1\lambda_2\right)^2 + \left(y'_1\lambda_1\right)^2}{\left(y'_1\lambda_1\right)^2}}} = \frac{\dfrac{\lambda_2}{y'_1}}{\dfrac{\sqrt{\left(x'_1\lambda_2\right)^2 + \left(y'_1\lambda_1\right)^2}}{y'_1\lambda_1}} = \frac{\lambda_2\lambda_1}{\sqrt{\left(x'_1\lambda_2\right)^2 + \left(y'_1\lambda_1\right)^2}}$$

Since we know that $x_1' = (\cos\phi)\sqrt{\lambda_1}$ and that $y_1' = (\sin\phi)\sqrt{\lambda_2}$,

$$P = \frac{\lambda_2\lambda_1}{\sqrt{\left(\cos^2\phi\right)\lambda_1\lambda_2^2 + \left(\sin^2\phi\right)\lambda_2\lambda_1^2}} = \frac{1}{\sqrt{\dfrac{\left(\cos^2\phi\right)\lambda_1\lambda_2^2 + \left(\sin^2\phi\right)\lambda_2\lambda_1^2}{\left(\lambda_2\lambda_1\right)^2}}} = \frac{1}{\sqrt{\dfrac{\cos^2\phi}{\lambda_1} + \dfrac{\sin^2\phi}{\lambda_2}}}$$

Now, to evaluate Ψ, we recall that $\sin^2\theta + \cos^2\theta = 1$:

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2\theta} \quad \text{or} \quad \tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$$\text{since} \quad \cos\Psi = \frac{P}{\sqrt{\lambda}}, \quad \frac{1}{\cos^2\Psi} = \frac{\lambda}{P^2} \quad \text{and} \quad \tan^2\Psi = \frac{\lambda}{P^2} - 1$$

$$\text{Consequently,} \quad \tan^2\Psi = \lambda\left(\frac{\cos^2\phi}{\lambda_1} + \frac{\sin^2\phi}{\lambda_2}\right) - 1$$

$$\text{Substituting for } \lambda, \quad \tan^2\Psi = \left(\lambda_1\cos^2\phi + \lambda_2\sin^2\phi\right)\left(\frac{\cos^2\phi}{\lambda_1} + \frac{\sin^2\phi}{\lambda_2}\right) - 1$$

$$\tan^2\Psi = \frac{\lambda_1}{\lambda_1}\cos^4\phi + \frac{\lambda_2}{\lambda_1}\sin^2\cos^2\phi + \frac{\lambda_1}{\lambda_2}\sin^2\cos^2\phi + \frac{\lambda_2}{\lambda_2}\sin^4\phi - 1$$

$$\tan^2\Psi = \cos^4\phi + \frac{\lambda_2}{\lambda_1}\sin^2\cos^2\phi + \frac{\lambda_1}{\lambda_2}\sin^2\cos^2\phi + \sin^4\phi - 1$$

$$\text{but} \quad 1 = \left(\sin^2\phi + \cos^2\phi\right)^2 = \sin^4\phi + 2\sin^2\phi\cos^2\phi + \cos^4\phi$$

$$\tan^2\Psi = \frac{\lambda_2}{\lambda_1}\sin^2\cos^2\phi + \frac{\lambda_1}{\lambda_2}\sin^2\cos^2\phi - 2\sin^2\phi\cos^2\phi$$

$$\text{so} \quad \tan^2\Psi = \left(\frac{\lambda_2}{\lambda_1} + \frac{\lambda_1}{\lambda_2} - 2\right)\sin^2\phi\cos^2\phi = \frac{\lambda_2^2 + \lambda_1^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2}\sin^2\phi\cos^2\phi = \frac{\left(\lambda_2 - \lambda_1\right)^2}{\lambda_1\lambda_2}\sin^2\phi\cos^2\phi$$

Taking the square roots of both sides leaves $\tan\Psi = \dfrac{\lambda_2 - \lambda_1}{\sqrt{\lambda_1\lambda_2}}\sin\phi\cos\phi$.

Once again, we'd prefer that our results were in terms of ϕ' instead of ϕ, so we'll again substitute:

$$\cos\phi = \frac{x_1}{\sqrt{\lambda_1}} = \frac{\sqrt{\lambda}\cos\phi'}{\sqrt{\lambda_1}}$$

$$\sin\phi = \frac{y_1}{\sqrt{\lambda_2}} = \frac{\sqrt{\lambda}\sin\phi'}{\sqrt{\lambda_2}}$$

$$\tan\Psi = \frac{\lambda_2 - \lambda_1}{\sqrt{\lambda_1\lambda_2}}\frac{\sqrt{\lambda}\cos\phi'}{\sqrt{\lambda_1}}\frac{\sqrt{\lambda}\sin\phi'}{\sqrt{\lambda_2}} = \lambda\frac{(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}\sin\phi'\cos\phi'$$

Recall that by definition, the shear strain, γ, is equal to tan Ψ. Then, for what will eventually be seen as a convenience, we will define an otherwise obscure parameter, γ ' as being equal to $\dfrac{\gamma}{\lambda}$. Combining this with our previously defined $\lambda' = \dfrac{1}{\lambda}$, etc., we arrive at:

$$\gamma' = (\lambda_1' - \lambda_2')\sin\phi'\cos\phi'$$

earlier we found that $\lambda' = \lambda_1'\cos^2\phi' + \lambda_2'\sin^2\phi$

Looks like a place to employ some double angle formulas. We remember that sin (a + b) = sin a cos b + sin b cos a, and that cos (a + b) = cos a cos b - sin a sin b, which leads to

$$\sin 2\phi' = 2\sin\phi'\cos\phi'$$

$$\cos 2\phi' = \cos^2\phi' - \sin^2\phi' = 1 - 2\sin^2\phi' = 2\cos^2\phi' - 1$$

$$\sin^2\phi' = \frac{1 - \cos 2\phi'}{2} \text{ and } \cos^2\phi' = \frac{1 + \cos 2\phi'}{2}$$

and we can substitute these into earlier results to obtain

$$\gamma' = \frac{(\lambda_1' - \lambda_2')}{2}\sin 2\phi'$$

$$\lambda' = \lambda_1'\frac{1 + \cos 2\phi'}{2} + \lambda_2'\frac{1 - \cos 2\phi'}{2} = \frac{(\lambda_1' + \lambda_2')}{2} + \frac{(\lambda_1' - \lambda_2')}{2}\cos 2\phi'$$

These are parametric equations for a circle in λ', γ' space, having a radius of $\dfrac{\lambda_1' - \lambda_2'}{2}$ and centered on the λ' axis at $\lambda' = \dfrac{\lambda_1' + \lambda_2'}{2}$